

Exemplar Problem

Conic Section

59. Equation of the hyperbola with eccentricity $\frac{3}{2}$ and foci at $(\pm 2, 0)$ is

(a) $\frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$

(b) $\frac{x^2}{9} - \frac{y^2}{9} = \frac{4}{9}$

(c) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(d) None of these

Ans:

Given that $e = \frac{3}{2}$

and foci = $(\pm ae, 0) = (\pm 2, 0)$

$\therefore ae = 2$

$$a \times \frac{3}{2} = 2 \Rightarrow a = \frac{4}{3}$$

Now we know that $b^2 = a^2 (e^2 - 1)$

$$b^2 = \frac{16}{9} \left(\frac{9}{4} - 1 \right) \Rightarrow b^2 = \frac{16}{9} \times \frac{5}{4}$$

$$\Rightarrow b^2 = \frac{20}{9}$$

So, the equation of the hyperbola is

$$\frac{x^2}{\left(\frac{4}{3}\right)^2} - \frac{y^2}{\left(\frac{20}{9}\right)} = 1$$

$$\Rightarrow \frac{9x^2}{16} - \frac{9y^2}{20} = 1 \Rightarrow \frac{x^2}{16} - \frac{y^2}{20} = \frac{1}{9}$$

$$\Rightarrow \frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$$

Hence, the correct option is (a).