31.Show that the set of all points such that the difference of their distances from $(4, 0)$ and $(-4, 0)$ is always equal to 2 represents a hyperbola.

Ans:

Given Two points $(4, 0)$and $(-4, 0)$.

The difference of distances of a point from the given two points is always 2 units.

Let the required point be $P\ (x, y)$.

Using the distance formula the distance between P and $(4, 0)$ is

$$= \sqrt{(x-4)^2 + (y-0)^2}$$

$$= \sqrt{(x-4)^2 + y^2}$$

Similarly, the distance between P and $(-4, 0)$ is

$$= \sqrt{(x-(-4))^2 + (y-0)^2}$$

$$= \sqrt{(x+4)^2 + y^2}$$

It is given that the difference of the distances is 2 units, so taking the difference,

$$\Rightarrow \sqrt{(x-4)^2 + y^2} - \sqrt{(x+4)^2 + y^2} = 2$$

$$\Rightarrow \sqrt{(x-4)^2 + y^2} = 2 + \sqrt{(x+4)^2 + y^2}$$

On squaring both the sides,

$$\Rightarrow (x-4)^2+y^2=4+(x+4)^2+y^2+4\sqrt{(x+4)^2+y^2}$$

$$\Rightarrow x^2+16-8x+y^2=4+x^2+16+8x+y^2+4\sqrt{(x+4)^2+y^2}$$

$$\Rightarrow \sqrt{(x+4)^2+y^2}=-(1+4x)$$

Again squaring both the sides,

$$\Rightarrow (x+4)^2+y^2=(-(1+4x))^2$$

$$\Rightarrow x^2+16+8x+y^2=1+16x^2+8x$$

$$\Rightarrow 15x^2-y^2=15$$

Hence, it can be seen that the above equation represents a hyperbola.