

## Exemplar Problem

### Conic Section

57. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half of the distance between the foci is

(a)  $\frac{4}{3}$

(b)  $\frac{4}{\sqrt{3}}$

(c)  $\frac{2}{\sqrt{3}}$

(d) None of these

**Ans:**

Length of the latus rectum of the hyperbola

$$= \frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a \dots\dots\dots (i)$$

Distance between the foci =  $2ae$

Transverse axis =  $2a$

and Conjugate axis =  $2b$

$$\therefore \frac{1}{2}(2ae) = 2b \Rightarrow ae = 2b \Rightarrow b = \frac{ae}{2} \dots (ii)$$

$$\Rightarrow b^2 = \frac{a^2 e^2}{4}$$

$$\Rightarrow 4a = \frac{a^2 e^2}{4} \text{ [from eq. (i)]}$$

$$\Rightarrow 16 = a e^2$$

$$\therefore a = \frac{16}{e^2}$$

$$\therefore \text{Now } b^2 = a^2 (e^2 - 1)$$

$$\Rightarrow 4a = a^2 (e^2 - 1)$$

$$\Rightarrow \frac{4}{a} = e^2 - 1 \Rightarrow \frac{4}{16/e} = e^2 - 1$$

$$\Rightarrow \frac{e^2}{4} = e^2 - 1 \Rightarrow e^2 - \frac{e^2}{4} = 1$$

$$\Rightarrow \frac{3e^2}{4} = 1 \Rightarrow e^2 = \frac{4}{3}$$

$$\therefore e = \frac{2}{\sqrt{3}}$$

$\therefore$  Hence, the correct option is (c).