

Exemplar Problem

Conic Section

22. Find the equation of the hyperbola with eccentricity $3/2$ and foci at $(\pm 2, 0)$.

Solution:

Given

$$e = \frac{3}{2}$$

We have foci $= (\pm a e, 0) = (\pm 2, 0)$

Therefore the hyperbola lies on x - axis,

$$\text{Equation is } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Given $a e = 2$

$$a \times \frac{3}{2} = 2$$

$$a = \frac{4}{3}$$

$$\therefore b^2 = a^2 (e^2 - 1)$$

$$b^2 = \left(\frac{4}{3}\right)^2 \left(\left(\frac{3}{2}\right)^2 - 1\right)$$

$$= \frac{16}{9} \left(\frac{9}{4} - 1\right) = \frac{16}{9} \times \frac{5}{4} = \frac{20}{9}$$

$$\text{Equation is } \frac{x^2}{\left(\frac{4}{3}\right)^2} - \frac{y^2}{\frac{20}{9}} = 1$$

$$\frac{9x^2}{16} - \frac{9y^2}{20} = 1$$

$$\text{Hence, the required equation is } \frac{9x^2}{16} - \frac{9y^2}{20} = 1$$

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