

Exemplar Problem

Conic Section

20. If the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then obtain the equation of the hyperbola.

Solution:

We know that equation of Hyperbola

$$= \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Also we have foci = $(\pm a e, 0)$

Given distance between foci is $2ae = 16$

$$e = \sqrt{2}$$

$$2 \times a \times \sqrt{2} = 16$$

$$a = \frac{16}{2 \times \sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

$$\therefore b^2 = a^2 (e^2 - 1)$$

$$b^2 = (4\sqrt{2})^2 ((\sqrt{2})^2 - 1)$$
$$= 32(2 - 1) = 32$$

$$\therefore \text{Equation is } \frac{x^2}{32} - \frac{y^2}{32} = 1$$