Write the first five terms of each of the sequences in Exercises 11 to 13 and obtain the corresponding series:

11. $a_1 = 3, a_n = 3a_{n-1} + 2$ for all $n > 1$

12. $a_1 = -1, a_n = \dfrac{a_{n-1}}{n}, n \geq 2$

13. $a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$

14. The Fibonacci sequence is defined by

$$1 = a_1 = a_2 \text{ and } a_n = a_{n-1} + a_{n-2}, n > 2.$$

Find $\dfrac{a_{n+1}}{a_n}$, for $n = 1, 2, 3, 4, 5$

9.4 Arithmetic Progression (A.P.)

Let us recall some formulae and properties studied earlier.

A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called *arithmetic sequence or arithmetic progression* if $a_{n+1} = a_n + d, n \in \mathbf{N}$, where a_1 is called the *first term* and the constant term d is called the *common difference* of the A.P.

Let us consider an A.P. (in its standard form) with first term a and common difference d, i.e., $a, a + d, a + 2d, \ldots$

Then the n^{th} term (*general term*) of the A.P. is $a_n = a + (n - 1)\, d$.

We can verify the following simple properties of an A.P. :

(i) If a constant is added to each term of an A.P., the resulting sequence is also an A.P.

(ii) If a constant is subtracted from each term of an A.P., the resulting sequence is also an A.P.

(iii) If each term of an A.P. is multiplied by a constant, then the resulting sequence is also an A.P.

(iv) If each term of an A.P. is divided by a non-zero constant then the resulting sequence is also an A.P.

Here, we shall use the following notations for an arithmetic progression:

a = the first term, l = the last term, d = common difference,

n = the number of terms.

S_n = the sum to n terms of A.P.

Let $a, a + d, a + 2d, \ldots, a + (n - 1)\, d$ be an A.P. Then

$$l = a + (n - 1)\, d$$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

We can also write, $S_n = \frac{n}{2}\left[a+l\right]$

Let us consider some examples.

Example 4 In an A.P. if m^{th} term is n and the n^{th} term is m, where $m \neq n$, find the pth term.

Solution We have $a_m = a + (m - 1) d = n$, ... (1)

and $a_n = a + (n - 1) d = m$ ... (2)

Solving (1) and (2), we get

$(m - n) d = n - m$, or $d = -1$, ... (3)

and $a = n + m - 1$ ... (4)

Therefore $a_p = a + (p - 1)d$

$= n + m - 1 + (p - 1)(-1) = n + m - p$

Hence, the p^{th} term is $n + m - p$.

Example 5 If the sum of n terms of an A.P. is $nP+\frac{1}{2}n(n-1)Q$, where P and Q are constants, find the common difference.

Solution Let $a_1, a_2, \ldots a_n$ be the given A.P. Then

$$S_n = a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_n = nP + \frac{1}{2}n(n-1)Q$$

Therefore $S_1 = a_1 = P$, $S_2 = a_1 + a_2 = 2P + Q$

So that $a_2 = S_2 - S_1 = P + Q$

Hence, the common difference is given by $d = a_2 - a_1 = (P + Q) - P = Q$.

Example 6 The sum of n terms of two arithmetic progressions are in the ratio $(3n + 8) : (7n + 15)$. Find the ratio of their 12^{th} terms.

Solution Let a_1, a_2 and d_1, d_2 be the first terms and common difference of the first and second arithmetic progression, respectively. According to the given condition, we have

$$\frac{\text{Sum to } n \text{ terms of first A.P.}}{\text{Sum to } n \text{ terms of second A.P.}} = \frac{3n+8}{7n+15}$$

or
$$\frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]}=\frac{3n+8}{7n+15}$$

or
$$\frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2}=\frac{3n+8}{7n+15} \qquad \dots (1)$$

Now
$$\frac{12^{\text{th}}\text{ term of first A.P.}}{12^{\text{th}}\text{ term of second A.P}}=\frac{a_1+11d_1}{a_2+11d_2}$$

$$\frac{2a_1+22d_1}{2a_2+22d_2}=\frac{3\times 23+8}{7\times 23+15} \qquad \text{[By putting } n = 23 \text{ in (1)]}$$

Therefore
$$\frac{a_1+11d_1}{a_2+11d_2}=\frac{12^{\text{th}}\text{ term of first A.P.}}{12^{\text{th}}\text{ term of second A.P.}}=\frac{7}{16}$$

Hence, the required ratio is 7 : 16.

Example 7 The income of a person is Rs. 3,00,000, in the first year and he receives an increase of Rs.10,000 to his income per year for the next 19 years. Find the total amount, he received in 20 years.

Solution Here, we have an A.P. with a = 3,00,000, d = 10,000, and n = 20.
Using the sum formula, we get,

$$S_{20}=\frac{20}{2}[600000+19\times 10000]=10\,(790000)=79{,}00{,}000.$$

Hence, the person received Rs. 79,00,000 as the total amount at the end of 20 years.

9.4.1 ***Arithmetic mean*** Given two numbers a and b. We can insert a number A between them so that a, A, b is an A.P. Such a number A is called the *arithmetic mean* (A.M.) of the numbers a and b. Note that, in this case, we have

$$A - a = b - A, \quad \text{i.e., } A = \frac{a+b}{2}$$

We may also interpret the A.M. between two numbers a and b as their average $\frac{a+b}{2}$. For example, the A.M. of two numbers 4 and 16 is 10. We have, thus constructed an A.P. 4, 10, 16 by inserting a number 10 between 4 and 16. The natural question now arises : Can we insert two or more numbers between given two numbers so that the resulting sequence comes out to be an A.P. ? Observe that two numbers 8 and 12 can be inserted between 4 and 16 so that the resulting sequence 4, 8, 12, 16 becomes an A.P.

More generally, given any two numbers a and b, we can insert as many numbers as we like between them such that the resulting sequence is an A.P.

Let $A_1, A_2, A_3, \ldots, A_n$ be n numbers between a and b such that $a, A_1, A_2, A_3, \ldots, A_n, b$ is an A.P.

Here, b is the $(n + 2)^{\text{th}}$ term, i.e., $b = a + [(n + 2) - 1]d = a + (n + 1) d$.

This gives $$d = \frac{b-a}{n+1}.$$

Thus, n numbers between a and b are as follows:

$$A_1 = a + d = a + \frac{b-a}{n+1}$$

$$A_2 = a + 2d = a + \frac{2(b-a)}{n+1}$$

$$A_3 = a + 3d = a + \frac{3(b-a)}{n+1}$$

....

....

$$A_n = a + nd = a + \frac{n(b-a)}{n+1}.$$

Example 8 Insert 6 numbers between 3 and 24 such that the resulting sequence is an A.P.

Solution Let A_1, A_2, A_3, A_4, A_5 and A_6 be six numbers between 3 and 24 such that

$3, A_1, A_2, A_3, A_4, A_5, A_6, 24$ are in A.P. Here, $a = 3$, $b = 24$, $n = 8$.

Therefore, $24 = 3 + (8 - 1) d$, so that $d = 3$.

Thus $A_1 = a + d = 3 + 3 = 6$; $A_2 = a + 2d = 3 + 2 \times 3 = 9$;

$A_3 = a + 3d = 3 + 3 \times 3 = 12$; $A_4 = a + 4d = 3 + 4 \times 3 = 15$;

$A_5 = a + 5d = 3 + 5 \times 3 = 18$; $A_6 = a + 6d = 3 + 6 \times 3 = 21$.

Hence, six numbers between 3 and 24 are 6, 9, 12, 15, 18 and 21.

EXERCISE 9.2

1. Find the sum of odd integers from 1 to 2001.
2. Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.
3. In an A.P., the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is –112.
4. How many terms of the A.P. $-6, -\frac{11}{2}, -5, \ldots$ are needed to give the sum –25?
5. In an A.P., if p^{th} term is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, prove that the sum of first pq terms is $\frac{1}{2}(pq+1)$, where $p \neq q$.
6. If the sum of a certain number of terms of the A.P. 25, 22, 19, … is 116. Find the last term.
7. Find the sum to n terms of the A.P., whose k^{th} term is $5k + 1$.
8. If the sum of n terms of an A.P. is $(pn + qn^2)$, where p and q are constants, find the common difference.
9. The sums of n terms of two arithmetic progressions are in the ratio $5n + 4 : 9n + 6$. Find the ratio of their 18^{th} terms.
10. If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first $(p + q)$ terms.
11. Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.

    Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$
12. The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of m^{th} and n^{th} term is $(2m - 1) : (2n - 1)$.
13. If the sum of n terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, find the value of m.
14. Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.
15. If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A.M. between a and b, then find the value of n.
16. Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A. P. and the ratio of 7^{th} and $(m - 1)^{\text{th}}$ numbers is 5 : 9. Find the value of m.

17. A man starts repaying a loan as first instalment of Rs. 100. If he increases the instalment by Rs 5 every month, what amount he will pay in the 30^{th} instalment?

18. The difference between any two consecutive interior angles of a polygon is $5°$. If the smallest angle is $120°$, find the number of the sides of the polygon.

9.5 Geometric Progression (G. P.)

Let us consider the following sequences:

$$\text{(i) } 2,4,8,16,\ldots, \text{ (ii) } \frac{1}{9}, \frac{-1}{27}, \frac{1}{81}, \frac{-1}{243} \ldots \text{ (iii) } .01, .0001, .000001, \ldots$$

In each of these sequences, how their terms progress? We note that each term, except the first progresses in a definite order.

In (i), we have $a_1 = 2, \frac{a_2}{a_1} = 2, \frac{a_3}{a_2} = 2, \frac{a_4}{a_3} = 2$ and so on.

In (ii), we observe, $a_1 = \frac{1}{9}, \frac{a_2}{a_1} = -\frac{1}{3}, \frac{a_3}{a_2} = -\frac{1}{3}, \frac{a_4}{a_3} = -\frac{1}{3}$ and so on.

Similarly, state how do the terms in (iii) progress? It is observed that in each case, every term except the first term bears a constant ratio to the term immediately preceding it. In (i), this constant ratio is 2; in (ii), it is $-\frac{1}{3}$ and in (iii), the constant ratio is 0.01. Such sequences are called *geometric sequence* or *geometric progression* abbreviated as G.P.

A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called *geometric progression*, if each term is non-zero and $\frac{a_{k+1}}{a_k} = r$ (constant), for $k \geq 1$.

By letting $a_1 = a$, we obtain a geometric progression, $a, ar, ar^2, ar^3, \ldots.$, where a is called the *first term* and r is called the *common ratio* of the G.P. Common ratio in geometric progression (i), (ii) and (iii) above are 2, $-\frac{1}{3}$ and 0.01, respectively.

As in case of arithmetic progression, the problem of finding the n^{th} term or sum of n terms of a geometric progression containing a large number of terms would be difficult without the use of the formulae which we shall develop in the next Section. We shall use the following notations with these formulae:

a = the first term, r = the common ratio, l = the last term,

n = the numbers of terms,

n = the numbers of terms,

S_n = the sum of first n terms.

9.5.1 General term of a G.P. Let us consider a G.P. with first non-zero term 'a' and common ratio 'r'. Write a few terms of it. The second term is obtained by multiplying a by r, thus $a_2 = ar$. Similarly, third term is obtained by multiplying a_2 by r. Thus, $a_3 = a_2 r = ar^2$, and so on.

We write below these and few more terms.

1st term = $a_1 = a = ar^{1-1}$, 2nd term = $a_2 = ar = ar^{2-1}$, 3rd term = $a_3 = ar^2 = ar^{3-1}$
4th term = $a_4 = ar^3 = ar^{4-1}$, 5th term = $a_5 = ar^4 = ar^{5-1}$
Do you see a pattern? What will be 16th term?

$$a_{16} = ar^{16-1} = ar^{15}$$

Therefore, the pattern suggests that the nth term of a G.P. is given by

$a_n = ar^{n-1}$.

Thus, a, G.P. can be written as $a, ar, ar^2, ar^3, \dots ar^{n-1}$; $a, ar, ar^2, \dots, ar^{n-1} \dots$;according as G.P. is *finite* or *infinite*, respectively.

The series $a + ar + ar^2 + \dots + ar^{n-1}$ or $a + ar + ar^2 + \dots + ar^{n-1} + \dots$ are called *finite* or *infinite geometric series*, respectively.

9.5.2. Sum to n terms of a G.P. Let the first term of a G.P. be a and the common ratio be r. Let us denote by S_n the sum to first n terms of G.P. Then

$$S_n = a + ar + ar^2 + \dots + ar^{n-1} \qquad \dots (1)$$

Case 1 If $r = 1$, we have $S_n = a + a + a + \dots + a$ (n terms) $= na$

Case 2 If $r \neq 1$, multiplying (1) by r, we have

$$rS_n = ar + ar^2 + ar^3 + \dots + ar^n \qquad \dots (2)$$

Subtracting (2) from (1), we get $(1 - r)\, S_n = a - ar^n = a(1 - r^n)$

This gives $\quad S_n = \dfrac{a(1-r^n)}{1-r} \quad$ or $\quad S_n = \dfrac{a(r^n-1)}{r-1}$

Example 9 Find the 10th and nth terms of the G.P. 5, 25,125,… .

Solution Here $a = 5$ and $r = 5$. Thus, $a_{10} = 5(5)^{10-1} = 5(5)^9 = 5^{10}$
and $\quad a_n = ar^{n-1} = 5(5)^{n-1} = 5^n$.

Example10 Which term of the G.P., 2,8,32, ... up to n terms is 131072?

Solution Let 131072 be the nth term of the given G.P. Here $a = 2$ and $r = 4$.
Therefore $\quad 131072 = a_n = 2(4)^{n-1} \quad$ or $\quad 65536 = 4^{n-1}$

This gives $\quad 4^8 = 4^{n-1}$.

So that $n - 1 = 8$, i.e., $n = 9$. Hence, 131072 is the 9th term of the G.P.

Example11 In a G.P., the 3rd term is 24 and the 6th term is 192.Find the 10th term.

Solution Here, $a_3 = ar^2 = 24$ ... (1)

and $a_6 = ar^5 = 192$ ... (2)

Dividing (2) by (1), we get $r = 2$. Substituting $r = 2$ in (1), we get $a = 6$.

Hence $a_{10} = 6\ (2)^9 = 3072$.

Example12 Find the sum of first n terms and the sum of first 5 terms of the geometric series $1+\frac{2}{3}+\frac{4}{9}+...$

Solution Here $a = 1$ and $r = \frac{2}{3}$. Therefore

$$S_n = \frac{a\,(1-r^n)}{1-r} = \frac{\left[1-\left(\frac{2}{3}\right)^n\right]}{1-\frac{2}{3}} = 3\left[1-\left(\frac{2}{3}\right)^n\right]$$

In particular, $S_5 = 3\left[1-\left(\frac{2}{3}\right)^5\right] = 3\times\frac{211}{243} = \frac{211}{81}$.

Example 13 How many terms of the G.P. $3, \frac{3}{2}, \frac{3}{4}, ...$ are needed to give the sum $\frac{3069}{512}$?

Solution Let n be the number of terms needed. Given that $a = 3$, $r = \frac{1}{2}$ and $S_n = \frac{3069}{512}$

Since $$S_n = \frac{a\,(1-r^n)}{1-r}$$

Therefore $$\frac{3069}{512} = \frac{3(1-\frac{1}{2^n})}{1-\frac{1}{2}} = 6\left(1-\frac{1}{2^n}\right)$$

or $$\frac{3069}{3072} = 1-\frac{1}{2^n}$$

or $$\frac{1}{2^n} = 1-\frac{3069}{3072} = \frac{3}{3072} = \frac{1}{1024}$$

or $$2^n = 1024 = 2^{10}, \text{ which gives } n = 10.$$

Example 14 The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is – 1. Find the common ratio and the terms.

Solution Let $\frac{a}{r}$, a, ar be the first three terms of the G.P. Then

$$\frac{a}{r} + ar + a = \frac{13}{12} \qquad \text{... (1)}$$

and $$\left(\frac{a}{r}\right)(a)\,(ar) = -1 \qquad \text{... (2)}$$

From (2), we get $a^3 = -1$, i.e., $a = -1$ (considering only real roots)

Substituting $a = -1$ in (1), we have

$$-\frac{1}{r} - 1 - r = \frac{13}{12} \text{ or } 12r^2 + 25r + 12 = 0.$$

This is a quadratic in r, solving, we get $r = -\frac{3}{4}$ or $-\frac{4}{3}$.

Thus, the three terms of G.P. are : $\frac{4}{3}, -1, \frac{3}{4}$ for $r = \frac{-3}{4}$ and $\frac{3}{4}, -1, \frac{4}{3}$ for $r = \frac{-4}{3}$,

Example15 Find the sum of the sequence 7, 77, 777, 7777, ... to n terms.

Solution This is not a G.P., however, we can relate it to a G.P. by writing the terms as

$$S_n = 7 + 77 + 777 + 7777 + \ldots \text{ to } n \text{ terms}$$

$$= \frac{7}{9}[9+99+999+9999+ \ldots\text{to } n \text{ term}]$$

$$= \frac{7}{9}[(10-1)+(10^2-1)+(10^3-1)+(10^4-1)+\ldots n \text{ terms}]$$

$$= \frac{7}{9}[(10+10^2+10^3+...n \text{ terms})-(1+1+1+...n \text{ terms})]$$

$$= \frac{7}{9}\left[\frac{10(10^n-1)}{10-1}-n\right] = \frac{7}{9}\left[\frac{10\,(10^n-1)}{9}-n\right].$$

Example 16 A person has 2 parents, 4 grandparents, 8 great grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own.

Solution Here $a = 2$, $r = 2$ and $n = 10$

Using the sum formula $\quad S_n = \dfrac{a\,(r^n-1)}{r-1}$

We have $\quad S_{10} = 2(2^{10} - 1) = 2046$

Hence, the number of ancestors preceding the person is 2046.

9.5.3 *Geometric Mean (G.M.)* The geometric mean of two positive numbers a and b is the number $\sqrt{ab}$. Therefore, the geometric mean of 2 and 8 is 4. We observe that the three numbers 2,4,8 are consecutive terms of a G.P. This leads to a generalisation of the concept of geometric means of two numbers.

Given any two positive numbers a and b, we can insert as many numbers as we like between them to make the resulting sequence in a G.P.

Let $G_1, G_2,\ldots, G_n$ be n numbers between positive numbers a and b such that $a, G_1, G_2, G_3,\ldots, G_n, b$ is a G.P. Thus, b being the $(n + 2)^{th}$ term,we have

$$b = ar^{n+1}, \quad \text{or} \quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}.$$

Hence $\quad G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}, \quad G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}, \quad G_3 = ar^3 = a\left(\frac{b}{a}\right)^{\frac{3}{n+1}},$

$$G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

Example17 Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.

Solution Let G_1, G_2, G_3 be three numbers between 1 and 256 such that $1, G_1, G_2, G_3, 256$ is a G.P.

Therefore $256 = r^4$ giving $r = \pm 4$ (Taking real roots only)

For $r = 4$, we have $G_1 = ar = 4$, $G_2 = ar^2 = 16$, $G_3 = ar^3 = 64$

Similarly, for $r = -4$, numbers are $-4, 16$ and -64.

Hence, we can insert 4, 16, 64 between 1 and 256 so that the resulting sequences are in G.P.

9.6 Relationship Between A.M. and G.M.

Let A and G be A.M. and G.M. of two given positive real numbers a and b, respectively. Then

$$A = \frac{a+b}{2} \text{ and } G = \sqrt{ab}$$

Thus, we have

$$A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2}$$

$$= \frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{2} \geq 0 \qquad \ldots (1)$$

From (1), we obtain the relationship $A \geq G$.

Example 18 If A.M. and G.M. of two positive numbers a and b are 10 and 8, respectively, find the numbers.

Solution Given that $\quad \text{A.M.} = \frac{a+b}{2} = 10 \qquad \ldots (1)$

and $\quad \text{G.M.} = \sqrt{ab} = 8 \qquad \ldots (2)$

From (1) and (2), we get

$$a + b = 20 \qquad \ldots (3)$$

$$ab = 64 \qquad \ldots (4)$$

Putting the value of a and b from (3), (4) in the identity $(a - b)^2 = (a + b)^2 - 4ab$, we get

$$(a - b)^2 = 400 - 256 = 144$$

or $\quad a - b = \pm 12$

... (5)

Solving (3) and (5), we obtain

$$a = 4, b = 16 \text{ or } a = 16, b = 4$$

Thus, the numbers a and b are 4, 16 or 16, 4 respectively.

EXERCISE 9.3

1. Find the 20th and n^{th} terms of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

2. Find the 12^{th} term of a G.P. whose 8^{th} term is 192 and the common ratio is 2.

3. The 5^{th}, 8^{th} and 11^{th} terms of a G.P. are p, q and s, respectively. Show that $q^2 = ps$.

4. The 4^{th} term of a G.P. is square of its second term, and the first term is -3. Determine its 7^{th} term.

5. Which term of the following sequences:

   (a) $2, 2\sqrt{2}, 4, \ldots$ is 128 ? (b) $\sqrt{3}, 3, 3\sqrt{3}, \ldots$ is 729 ?

   (c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ is $\frac{1}{19683}$?

6. For what values of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P.?

Find the sum to indicated number of terms in each of the geometric progressions in Exercises 7 to 10:

7. 0.15, 0.015, 0.0015, ... 20 terms.

8. $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \ldots$ n terms.

9. $1, -a, a^2, -a^3, \ldots$ n terms (if $a \neq -1$).

10. $x^3, x^5, x^7, \ldots$ n terms (if $x \neq \pm 1$).

11. Evaluate $\sum_{k=1}^{11} (2+3^k)$.

12. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

13. How many terms of G.P. $3, 3^2, 3^3, \ldots$ are needed to give the sum 120?

14. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

15. Given a G.P. with $a = 729$ and 7^{th} term 64, determine S_7.

16. Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term.

17. If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y and z, respectively. Prove that x, y, z are in G.P.

18. Find the sum to n terms of the sequence, 8, 88, 888, 8888… .

19. Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, $\frac{1}{2}$.

20. Show that the products of the corresponding terms of the sequences $a, ar, ar^2, \ldots ar^{n-1}$ and A, AR, AR^2, … AR^{n-1} form a G.P, and find the common ratio.

21. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4th by 18.

22. If the p^{th}, q^{th} and r^{th} terms of a G.P. are a, b and c, respectively. Prove that

$$a^{q-r}\, b^{r-p} c^{P-q} = 1.$$

23. If the first and the n^{th} term of a G.P. are a and b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

24. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n + 1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

25. If a, b, c and d are in G.P. show that

$(a^2 + b^2 + c^2)\,(b^2 + c^2 + d^2) = (ab + bc + cd)^2$.

26. Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

27. Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

28. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

29. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

30. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and n^{th} hour ?

31. What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

32. If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.