A.M. – G.M. – H.M. INEQUALITY

$x_1, x_2, x_3, \ldots\ldots\ldots, x_n \rightarrow$ All positive numbers

$$\text{Arithmetic Mean (A.M.)} = \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n}$$

$$\text{Geometric Mean (G.M.)} = (x_1 \cdot x_2 \cdot x_3 \cdot \ldots\ldots \cdot x_n)^{1/n}$$

$$\text{Harmonic Mean (H.M.)} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \ldots\ldots + \frac{1}{x_n}}$$

$$\text{A.M.} \geq \text{G.M.} \geq \text{H.M.}$$

Equality holds if all numbers are equal i.e. A.M. = G.M. = H.M.

If at least two numbers are unequal then A.M. > G.M. > H.M.

ARITHMETIC MEANS

$a, A_1, A_2, A_3, \ldots\ldots\ldots, A_n, b \rightarrow$ in A.P.

$A_i \rightarrow$ Arithmetic Means

$\Rightarrow A_1+A_2+\ldots\ldots\ldots+A_n = \frac{a+b}{2} \times n$

$\Rightarrow A_K = a+\left(\frac{b-a}{n+1}\right) K$

GEOMETRIC MEANS

$a, G_1, G_2, G_3, \ldots\ldots, G_n, b \rightarrow$ in G.P.

$G_i \rightarrow$ Geometric Means

$\Rightarrow G_1 \cdot G_2 \cdot G_3 \ldots\ldots\ldots G_n = \left(\sqrt{ab}\right)^n$

$\Rightarrow G_K = a.\left(\frac{b}{a}\right)^{\frac{K}{n+1}}$ $\quad G_K = K^{th}$ Geometric Mean

n^{th} TERM AND SUM OF N TERMS OF A.P., G.P., H.P.

	Arithmetic Progression (A.P.)	Geometric Progression (G.P.)	Harmonic Progression (H.P.)
	$a, a + d, a + 2d, \ldots$	$a, ar, ar^2, \ldots$	$a, b, \ldots$
n^{th} term (t_n)	$a + (n - 1)d$	$a \cdot r^{n-1}$	$\dfrac{1}{\frac{1}{a} + (n-1)\left(\frac{1}{b} - \frac{1}{a}\right)}$
Sum of n terms (S_n)	$\dfrac{n}{2}[2a + (n-1)d]$ OR $\dfrac{n}{2}[a + t_n]$	$\dfrac{a(1 - r^n)}{(1 - r)}$ $(r \neq 1)$ $S_\infty = \dfrac{a}{1-r}$ $(\lvert r \rvert < 1)$	Formula does not exist

Recognition of A.P., G.P., H.P.

If a, b, c are three consecutive terms

$\dfrac{a-b}{b-c} = \dfrac{a}{a}$	$\dfrac{a-b}{b-c} = \dfrac{a}{b}$	$\dfrac{a-b}{b-c} = \dfrac{a}{c}$
a, b, c are in A.P.	a, b, c are in G.P.	a, b, c are in H.P.