1. **The first term of an A.P.is a, and the sum of the first p terms is zero, show that the sum of its next q terms is $\frac{-a\,(p+q)\,q}{p-1}$.**

[Hint: Required sum = S $_{p+q}$ – S $_{p}$]

Solution:

Given first term is 'a' and sum of first p terms is $S_p = 0$
Now we have to find the sum of next q terms
Therefore, total terms are p + q
Hence, sum of all terms minus the sum of the first p terms will give the sum of next q terms
But sum of first p terms is 0 hence sum of next q terms will be the same as sum of all terms
So, we have to find sum of p + q terms
Sum of n terms of AP is given by $S_n = \left(\frac{n}{2}\right)(2a + (n-1)d)$
Where a is first term and d is the common difference
Using the given hint we get
⇒ required sum = $S_{p+q} - S_p$
Using S_n formula

$$\Rightarrow \text{required sum} = \frac{p+q}{2}(2a + (p+q-1)d) \quad \text{......1}$$

Now we have to find d
We use the given $S_p = 0$ to find d

$$\Rightarrow S_p = \frac{p}{2}(2a + (p-1)d)$$

⇒ 0 = 2a + (p – 1) d

$$\Rightarrow d = -\frac{2a}{p-1}$$

Put this value of d in 1 we get

$$\Rightarrow \text{required sum} = \frac{p+q}{2}\left(2a + (p+q-1)\left(-\frac{2a}{p-1}\right)\right)$$

Taking LCM and simplifying we get

$$= \frac{p+q}{2}\left(2a - \frac{2ap}{p-1} - \frac{2aq}{p-1} + \frac{2a}{p-1}\right)$$

On computing we get

$$= a(p+q)\left(1 - \frac{p}{p-1} - \frac{q}{p-1} + \frac{1}{p-1}\right)$$

$$= a(p+q)\left(1 + \frac{-(p-1)}{p-1} - \frac{q}{p-1}\right)$$

$$= a(p+q)\left(1 + (-1) - \frac{q}{p-1}\right)$$

$$= -\frac{a(p+q)q}{p-1}$$

Hence proved.