6.23 A conducting wire XY of mass m and neglibile resistance slides smoothly on two parallel conducting wires as shown in Fig 6.11. The closed circuit has a resistance R due to AC. AB and CD are perfect conductors. There is a magnetic field $\mathbf{B} = B(t)\hat{\mathbf{k}}$.

(i) Write down equation for the acceleration of the wire XY.

(ii) If **B** is independent of time, obtain $v(t)$, assuming $v(0) = u_0$.

(iii) For (b), show that the decrease in kinetic energy of XY equals the heat lost in R.

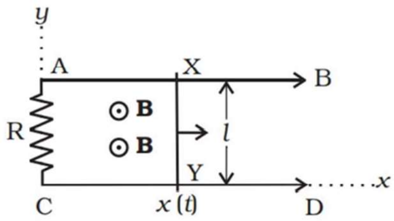

Fig. 6.11

Solution:

Ans: At t=0, let wire XY be at x=0.

And x=x is at t=t (t)

The magnetic flux is a time-dependent quantity.

$$\phi(t) = B(t) \times A$$

$$\phi(t) = B(t)\,[l.\,x(t)]$$

$$\varepsilon = -\frac{d\phi(t)}{dt} = -\frac{dB(t)}{dt} l.\,x(t) - B(t)l.\,\frac{dx(t)}{dt}$$

$$\varepsilon = -\frac{dB(t)}{dt} l.\,x(t) - B(t)lv(t)$$

In loop XYAX, the direction of induced current determined by Fleming's Right Hand Rule or Lenz's rule is clockwise.

$$\varepsilon = \frac{\varepsilon}{R} = \frac{-l}{R}\left[x(t)\frac{db(t)}{dt} + B(t)v(t)\right] \ldots\ldots\ldots\ldots\ldots(1)$$

The conductor is being acted upon by a force.

$$F = B(t)\ I\, l\, sin\, 90^\circ$$

$$F = B(t)\ I\, l$$

$$F = \frac{B(t)\ I\, l}{R} = \frac{-B(t)l^2}{R}\left[-\frac{dB}{dt}.\,x(t) - B(t).\,v(t)\right]$$

$$\frac{md^2x}{dt^2} = \frac{-B(t)l^2}{R}\left[-\frac{dB}{dt}.\,x(t) - B(t).\,v(t)\right] \ldots\ldots\ldots\ldots(2)$$

(ii) Assuming $v\ (0) = u_o$, find v(t) if B is time independent.

Ans: B is now time independent, i.e., it does not vary with time or remains constant.

$\frac{dB}{dt} = \mathbf{0}$, B(t)=B and v(t)=v ....(3)

Putting (3) in (2) we get

$$\frac{d^2x}{dt^2} = \frac{-l^2}{mR}[0 + B.v]$$

$$\frac{d^2x}{dt^2} + \frac{B^2l^2}{mR}\frac{dx}{dt} = 0$$

$$\frac{dv}{dt} + \frac{B^2l^2}{mR}v = 0$$

We can integrate using a variable separable from a differential equation.

$v = A\exp\left(\frac{-l^2B^2t}{mR}\right)$ at t=0, v=u

$$v(t) = u\exp\left(\frac{-l^2B^2t}{mR}\right)\ldots\ldots\ldots\ldots(4)$$

(iii) For (b), prove that the kinetic energy of XY decreases by the same amount as the heat lost in R.

Ans: Power consumption is represented as $P = I^2R$

As a result, the energy spent during the time interval dt is expressed as $= \frac{m}{2\,u_o{}^2} - - \frac{m}{2\,v^2\,(t)}$

The kinetic energy is decreasing, as shown by the equation above.

Therefore, energy consumed in time interval dt is given as $\frac{m}{2}u_0^2 - \frac{m}{2}v^2\,(t)$

The above equation shows that there is a decrease in the kinetic energy.