General and Middle Terms

1. In the binomial expansion for $(a+b)^n$, we observe that the first term is ${}^nC_0 a^n$, the second term is ${}^nC_1 a^{n-1} b$, the third term is ${}^nC_2 a^{n-2} b^2$, and so on. Looking at the pattern of the successive terms we can say that the $(r+1)^{th}$ term is ${}^nC_r a^{n-r} b^r$.
The $(r+1)^{th}$ term is also called the general term of the expansion $(a+b)^n$. It is denoted by T_{r+1}. Thus $T_{r+1} = {}^nC_r a^{n-r} b^r$.

2. Regarding the middle term in the expansion $(a+b)^n$, we have

   (i) If n is even, the middle term is ~~nth~~ $\left(\frac{n}{2}+1\right)^{th}$ term

   (ii) If n is odd, then $n+1$ is even. there will be two middle terms in the expansion, namely, $\left(\frac{n+1}{2}\right)^{th}$ term and $\left(\frac{n+1}{2}+1\right)^{th}$ term

3. In the expansion of $\left(x+\frac{1}{x}\right)^{2n}$, where $x \neq 0$, the middle term is $\left(\frac{2n+1+1}{2}\right)^{th}$

   i.e, $(n+1)^{th}$ term, as $2n$ is even.

   It is given by ${}^{2n}C_n x^n \left(\frac{1}{x}\right)^n = {}^{2n}C_n$ (constant).

   This term is called the term independent of x or the constant term

We can use derivatives of equations involving binomial expressions to easily solve problems.

Example:

$$f(x) = 1 - x + x^2 - x^3 + \dots + x^{16} - x^{17}$$

$$= a_0 + a_1(1+x) + \dots \quad a_{17}(1+x)^{17}$$

$$\Rightarrow f' = \frac{df}{dx} = -1 + 2x - 3x^2 + \dots + 16x^{15} - 17x^{16}$$

$$= a_1 + 2a_2(1+x) + 3a_3(1+x)^2 + 4a_4(1+x)^3 + \dots + 17a_{17}(1+x)^{16}.$$

This makes finding the value of 'a_i' easier.