Verify the following

1. $\int \frac{2x-1}{2x+3}dx = x - \log|(2x+3)^2| + C$

Solution:

L.H.S. $= \int \frac{2x-1}{2x+3}dx$

$= \int\left(1 - \frac{4}{2x+3}\right)dx$ [Dividing the numerator by the denominator]

$= \int 1.dx - 4\int \frac{1}{2x+3}dx = \int 1.dx - \frac{4}{2}\int \frac{1}{x+\frac{3}{2}}dx$

$= \int 1.dx - 2\int \frac{1}{x+\frac{3}{2}}dx = x - 2\log\left|x+\frac{3}{2}\right| + C$

$= x - 2\log\left|\frac{2x+3}{2}\right| + C = x - \log\left|\left(\frac{2x+3}{2}\right)^2\right| + C$

[$\because n\log m = \log m^n$]

$= x - \log|(2x+3)^2| - \log 2^2 + C$

$= x - \log|(2x+3)^2| + C_1 \Rightarrow$ R.H.S. [where $C_1 = C - \log 2^2$]

L.H.S. = R.H.S.

- Hence Proved