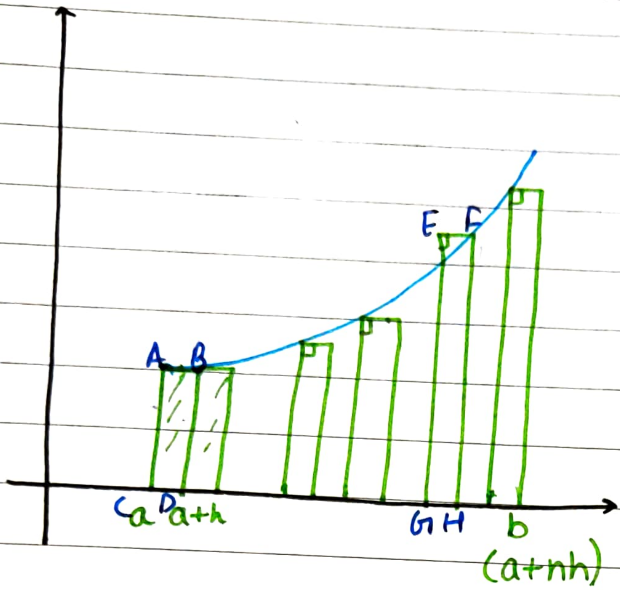
Reimann Sum (limit as a Sum)



$S_1 = hf(a) + hf(a+h) + \dots$

$$\boxed{S_1 = \sum_{r=0}^{n-1} f(a+rh) \;:\; h \to 0,\; n \to \infty}$$

also we if calc. area by making rect. like GHEF we get

$$\boxed{S_2 = \sum_{r=1}^{n} f(a+rh) : h \to 0,\; n \to \infty}$$

$*\; S_1 \leq S \leq S_2$ //.

these both are same as the graph is sandwiched

also, $\boxed{\frac{b-a}{n} = h}$

$*$ $a = 0$ $\quad b = 1$

we see, $$\int_0^1 f(x)\,dx = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} f\left(\frac{r}{n}\right)$$

we can convert the sum $\to$ integral

$\left\{ \frac{r}{n} \to x,\; \frac{1}{n} \to dx \right\}$

lower limit $= \frac{1}{n} = 0$

upper limit $\Rightarrow \frac{n}{n} = 1$ //

Q $\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n+1}+\cdots+\frac{1}{4n}\right)$

Ans $\lim_{n\to\infty}\sum_{r=0}^{3n}\frac{1}{n+r} = \sum_{r=0}^{3n}\frac{1/n}{1+\frac{r}{n}} = \int_0^3\frac{dx}{1+x}$

$= \ln 4$

Q $\lim_{n\to\infty}\left(\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+\cdots+\frac{3n}{5n^2}\right)$

$= \sum_{r=1}^{2n}\frac{n+r}{n^2+r^2} = \sum_{r=1}\frac{\frac{1}{n}\left(1+\frac{r}{n}\right)}{1+\left(\frac{r}{n}\right)^2}$

$= \int^2\frac{1+x}{1+x^2}\,dx$

Page No.:

Date:

youva

$$\int \underbrace{(\text{Algebric})}_{u}\underbrace{(\text{—})}_{v}\,dx$$

<u>use</u>.

$$\boxed{u v_1 - u' v_2 + u'' v_3 - u''' v_4 + - - -}$$

$$v_2 = \int v_1\,dx$$

$$v_1 = \int v\,dx$$

when $\int v$ is too easy

so//

$\boxed{T-1}$

① $\int \frac{dx}{a+b\sin^2 x}$ $\quad$ $\int \frac{dx}{a+b\cos^2 x}$ $\quad$ $\int \frac{dx}{a\cos^2 x + b\cos^2 x + c\sin x \cdot \cos x}$

divide $\cos^2 x$

then $\tan x \Rightarrow t$

② $\int \frac{dx}{a+b\cos x}$ $\quad$ $\int \frac{dx}{a+b\sin x}$ $\quad$ $\int \frac{dx}{a+b\sin x + c\cos x}$

use half angle formula

Date:

T-3.

$$\frac{x^2 \pm \lambda}{x^4 + kx^2 + \lambda^2}$$

divide by x^2.

λ

Integration containing Irrational f^n

T-1 $\displaystyle\int \frac{dx}{(ax+b)\sqrt{px+q}}$ $px+q=t^2$

T-2 $\displaystyle\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$ $ax+b=\frac{1}{t}$ then done

T-3 $\displaystyle\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$ $px+q=t^2$

T-4 $\displaystyle\int \frac{dx}{(ax^2+bx+c)\sqrt{px^2+qx+r}}$

T-4 Ques

① $ax^2 + bx + c = a(x-\alpha)(x-\beta)$

$$\int \frac{dx}{(x^2-x-2)\sqrt{x^2+x+1}} = A\int \frac{dx}{(x-2)\sqrt{x^2+x+1}} + B\int \frac{dx}{(x+1)(\sqrt{x^2+x+1})}$$

type 2.

② $ax^2+bx+c =$ perfect square $= (lx+m)^2$: put $lx+m = \frac{1}{t}$

③ $b=0$, $q=0$: put $x = \frac{1}{t}$ then put $\boxed{p+\cancel{q}t^2 = u^2}$

or

M-2 $\int \frac{dx}{(x^2+4)\sqrt{4x^2+4}}$

put $\left(\frac{4x^2+1}{x^2+4} = t^2\right) \Rightarrow x^2 = \frac{4t^2-1}{4-t^2}$

$x^2+4 = \frac{15}{4-t^2}$

$2x\frac{dx}{dt} = \frac{30t}{(4-t^2)^2}$

$4x^2+1 = \frac{15t^2}{4-t^2}$

Substituting