

Schwarz Inequality

if $f^2(x) + g^2(x)$ are integrable on $[a, b]$
then

$$\left| \int_a^b f(x)g(x) dx \right| \leq \sqrt{\left(\int_a^b f^2(x) dx \right) \left(\int_a^b g^2(x) dx \right)}$$

Proof.

$$f(x) = \int_a^b (f(x) - \lambda(g(x)))^2 \geq 0 : \lambda \in \mathbb{R}$$

$$\lambda^2 \int_a^b g^2(x) dx - 2\lambda \int_a^b f(x)g(x) dx$$

$$+ \int_a^b f^2(x) dx \geq 0 \quad \forall \lambda \in \mathbb{R}$$

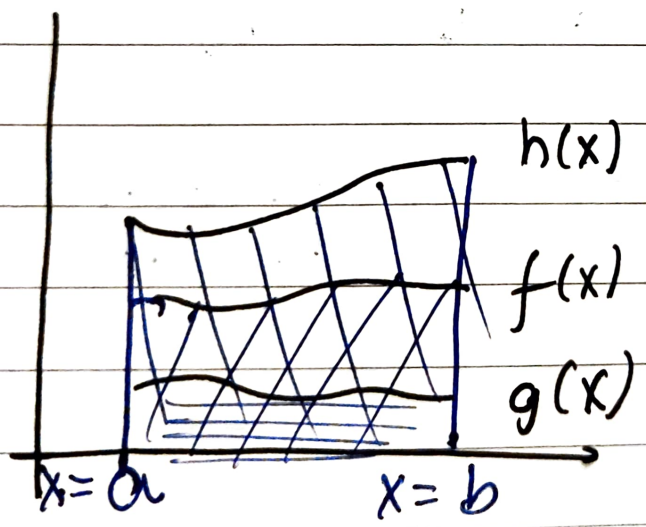
then $D \leq 0$

H. Proved

Inequality (estimation)

if $g(x) \leq f(x) \leq h(x)$ then

$$\int_a^b g(x) dx \leq \int_a^b f(x) dx \leq \int_a^b h(x) dx$$



Draw graph

find max. & min.

③

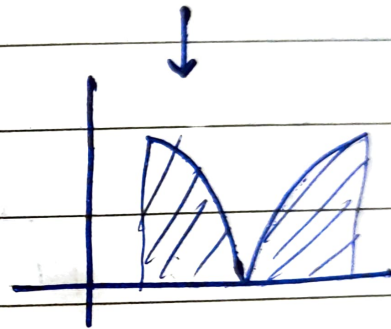
$$\left| \int_a^b f(x) dx \right| \leq \int_a^b |f(x)| dx$$

f(x)

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individual
sum ka
mod

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like $|5-3| < |5|+|3|$

$$\star \int_0^{\frac{\pi}{2}} \ln(\sin x) dx = \int_0^{\frac{\pi}{2}} \ln(\cos x) dx = -\frac{\pi}{2} \ln 2$$

$$\star \int_0^{\frac{\pi}{2}} \ln(\sec x) dx = \frac{\pi}{2} \ln 2 \quad \star \int_0^{\frac{\pi}{2}} \ln(\tan x) dx = 0$$