Question 3: The value of

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^x} dx$$

is equal to

(a) $\pi^2/4 - 2$

(b) $\pi^2/4 + 2$

(c) $\pi^2/4$

(d) 0

Answer: a

Solution:

Let I =

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^x} dx$$

=

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^{-x}} dx$$

I =

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{e^x x^2 \cos x}{1+e^x} dx$$

Since $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$2I = \int_{-\pi/2}^{\pi/2} x^2 \cos x\,dx = 2\int_0^{\pi/2} x^2 \cos x\,dx$

$I = [x^2 \sin x + 2x \cos x - 2 \sin x]_0^{\pi/2}$

$= \pi^2/4 - 2$

Hence option a is the answer.