Question 4: The value of $\int_0^1 4x^3 (d^2/dx^2)(1-x^2)^5 \, dx$ is

(a) 2

(b) 1

(c) 3

(d) 0

Answer: a

Solution:

Use integration by parts

$\int_0^1 4x^3 (d^2/dx^2)(1-x^2)^5 \, dx = [4x^3 (d/dx)(1-x^2)^5]_0^1 - \int_0^1 12x^2 (d/dx)(1-x^2)^5 \, dx$

$= [4x^3 \times 5(1-x^2)(-2x)]_0^1 - 12 [[x^2(1-x^2)^5]_0^1 - \int_0^1 2x(1-x^2)^5 \, dx]$

$= 0 - 0 - 12 (0-0) + 12 \int_0^1 2x(1-x^2)^5 \, dx]$

$= 12 \times [-(1-x^2)^6/6]_0^1$

$= 12(0+1/6)$

$= 2$

Hence option a is the answer.