Q Let A, B, C be $n \times n$ real matrices that are pairwise commutative, $ABC = O_n$, PT

$$\det(A^3+B^3+C^3)\det(A+B+C) \geq 0$$

Ans. $A^3+B^3+C^3-3ABC$

$$= (A+B+C)(A^2+B^2+C^2-AB-BC-CA)$$

$$= (A+B+C)(A+\omega B+\omega^2 C)(A+\omega B+\omega^2 C)$$

$\det(A+B+C)\ \det(LHS) = \det(A+B+C)^2 \det(\alpha)\overline{\det(\alpha)}$

$\cancel{\det(LHS) =}$ ≥ 0 H.P

★ Trace of a Matrix ⇒ Sum of elements of diagonal (Principal)