

## Exemplar Problem

### Trigonometry Functions

**73.** If  $\operatorname{cosec} x = 1 + \cot x$  then  $x = 2n\pi, 2n\pi + \frac{\pi}{2}$ .

**Ans:** Given,  $\operatorname{cosec} x = 1 + \cot x$

$$\Rightarrow \frac{1}{\sin x} = 1 + \frac{\cos x}{\sin x}$$

$$\Rightarrow \frac{1}{\sin x} = \frac{\sin x + \cos x}{\sin x}$$

$$\Rightarrow \sin x + \cos x = 1$$

Divide whole equation by  $\sqrt{2}$ .

$$\Rightarrow \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}}$$

We know that  $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$  and  $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ . Therefore, we can write above written equation as,

$$\Rightarrow \sin \frac{\pi}{4} \sin x + \cos \frac{\pi}{4} \cos x = \frac{1}{\sqrt{2}}$$

Or

$$\Rightarrow \cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

We know that  $\cos(x - y) = \cos x \cos y + \sin x \sin y$ . Therefore, we get

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos \frac{\pi}{4}$$

We know that if  $\cos \theta = \cos \alpha$ , then  $\theta = 2n\pi \pm \alpha$ . Therefore, we get

$$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{4} + \frac{\pi}{4} \text{ or } \Rightarrow x = 2n\pi - \frac{\pi}{4} + \frac{\pi}{4}$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{2} \text{ or } \Rightarrow x = 2n\pi$$

Thus, the given statement is true.