

Exemplar Problem

Trigonometry Functions

72. One value of θ which satisfies the equation $\sin^4 \theta - 2\sin^2 \theta - 1$ lies between 0 and 2π .

Ans: We have, $\sin^4 \theta - 2\sin^2 \theta - 1$

Let $y = \sin^2 \theta$. Therefore, we get

$$\Rightarrow y^2 - 2y - 1 = 0$$

We know that for quadratic equation $ax^2 + bx + c = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Therefore, we get

$$\Rightarrow y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times -1}}{2 \times 1}$$

$$\Rightarrow y = \frac{2 \pm \sqrt{4+4}}{2}$$

$$\Rightarrow y = \frac{2 \pm 2\sqrt{2}}{2} = \frac{2(1 \pm \sqrt{2})}{2}$$

On canceling common terms, we get

$$\Rightarrow y = 1 \pm \sqrt{2}$$

$$\Rightarrow \sin^2 \theta = 1 \pm \sqrt{2}$$

$$\Rightarrow \sin^2 \theta = 1 + \sqrt{2} \text{ or } \sin^2 \theta = 1 - \sqrt{2}$$

We know that $-1 \leq \sin \theta \leq 1$. Therefore, we say that $\sin^2 \theta \leq 1$.

But we have,

$$\Rightarrow \sin^2 \theta = 1 + \sqrt{2} \text{ or } \sin^2 \theta = 1 - \sqrt{2}$$

Which is not possible.

Thus, the given statement is false.