

Exemplar Problem

Trigonometric Functions

29. Find the general solution of the equation $(\sqrt{3} - 1) \cos \theta + (\sqrt{3} + 1) \sin \theta = 2$

[Hint: Put $\sqrt{3} - 1 = r \sin \alpha$, $\sqrt{3} + 1 = r \cos \alpha$ which gives $\tan \alpha = \tan((\pi/4) - (\pi/6))$ $\alpha = \pi/12$]

Solution:

Let, $r \sin \alpha = \sqrt{3} - 1$ and $r \cos \alpha = \sqrt{3} + 1$

Therefore, $r = \sqrt{(\sqrt{3} - 1)^2 + (\sqrt{3} + 1)^2} = \sqrt{8} = 2\sqrt{2}$

And, $\tan \alpha = (\sqrt{3} - 1) / (\sqrt{3} + 1)$

Therefore, $r(\sin \alpha \cos \theta + \cos \alpha \sin \theta) = 2$

$\Rightarrow r \sin (\theta + \alpha) = 2$

$\Rightarrow \sin (\theta + \alpha) = 1/\sqrt{2}$

$\Rightarrow \sin (\theta + \alpha) = \sin (\pi/4)$

$\Rightarrow \theta + \alpha = n\pi + (-1)^n (\pi/4), n \in \mathbb{Z}$

$\Rightarrow \theta = n\pi + (-1)^n (\pi/4) - (\pi/12), n \in \mathbb{Z}$