28. Find the general solution of the equation sin x – 3sin2x + sin3x = cos x – 3cos2x + cos3x

Solution:

According to the question,

sin x – 3sin2x + sin3x = cos x – 3cos2x + cos3x

Grouping sin x and sin 3x in LHS and, cos x and cos 3x in RHS,

We get,

sin x + sin3x – 3sin2x = cos x + cos3x – 3cos2x

Applying transformation formula,

cos A + cos B = 2cos ((A + B)/2) cos((A – B)/2)

sin A + sin B = 2sin ((A + B)/2) cos((A – B)/2)

⇒

$$2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - 3\sin 2x = 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - 3\cos 2x$$

⇒ 2sin 2x cos x – 3sin 2x = 2cos 2x cos x – 3cos 2x

⇒ 2sin 2x cos x – 3sin 2x – 2cos 2x cos x + 3cos 2x = 0

⇒ 2cos x (sin 2x – cos 2x) – 3(sin 2x – cos 2x) = 0

⇒ (sin 2x – cos 2x)(2cos x – 3) = 0

⇒ cos x = 3/2 or sin 2x = cos 2x

As cos x ∈ [-1,1]

Hence, no value of x exists for which cos x = 3/2

Therefore, sin 2x = cos 2x

⇒ tan 2x = 1 = tan π/4

We know solution of tan x = tan α is given by,

x= nπ + α , n ∈ Z

Therefore, 2x = nπ + (π/4)

⇒ x = nπ/2 + (π/8), n ∈ Z