

Exemplar Problem

Trigonometric Functions

27. Find the general solution of the equation

$$5\cos^2 \theta + 7\sin^2 \theta - 6 = 0$$

Solution:

According to the question,

$$5\cos^2 \theta + 7\sin^2 \theta - 6 = 0$$

We know that,

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\text{Therefore, } 5\cos^2 \theta + 7(1 - \cos^2 \theta) - 6 = 0$$

$$\Rightarrow 5\cos^2 \theta + 7 - 7\cos^2 \theta - 6 = 0$$

$$\Rightarrow -2\cos^2 \theta + 1 = 0$$

$$\Rightarrow \cos^2 \theta = \frac{1}{2}$$

$$\text{Therefore, } \cos \theta = \pm 1/\sqrt{2}$$

$$\text{Therefore, } \cos \theta = \cos \pi/4 \text{ or } \cos \theta = \cos 3\pi/4$$

Since, solution of $\cos x = \cos a$ is given by

$$x = 2m\pi \pm a \quad \forall m \in \mathbb{Z}$$

$$\theta = n\pi \pm \pi/4, n \in \mathbb{Z}$$