Trigonometric Functions - Class XI

Related Questions with Solutions

Questions

Ouetion: 01

Let α and β be non-zero real numbers such that $2(\cos\beta-\cos\alpha)+\cos\alpha\cos\beta=1$. Then which of the following is/are true?

A.
$$\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$$
B. $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$
C. $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$
D. $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

Solutions

Solution: 01

Let
$$\tan^2 \frac{\alpha}{2} = m$$
 and $\tan^2 \frac{\beta}{2} = n$, then
$$\cos \alpha = \frac{1-m}{1+m} \text{ and } \cos \beta = \frac{1-n}{1+n}$$

$$We \ have, \ 2(\cos \beta - \cos \alpha) + \cos \alpha \cos \beta = 1$$

$$\Rightarrow 2\left(\frac{1-n}{1+n} - \frac{1-m}{1+m}\right) + \frac{1-m}{1+m} \cdot \frac{1-n}{1+n} = 1$$

$$\Rightarrow 2\{(1+m)(1-n) - (1-m)(1+n)\} + (1-m)(1-n) = (1+m)(1+n)$$

$$\Rightarrow 2\{1+m-n-mn-1-n+m+mn\} + 1-m-n+mn = 1+m+n+mn$$

$$\Rightarrow 4(m-n) = 2(m+n) \Rightarrow 2m-2n = m+n \Rightarrow m = 3n$$

$$\Rightarrow \tan^2 \frac{\alpha}{2} = 3\tan^2 \frac{\beta}{2}$$
Thus, $\tan \frac{\alpha}{2} = \pm \sqrt{3} \tan \frac{\beta}{2}$

Correct Options

Answer:01

Correct Options: A, B