Concepts and Formulas

Trigonometric Functions

Trigonometric Functions of Sum or Difference of Two Angles

(a) $\sin(\mathrm{A}+\mathrm{B}) = \sin \mathrm{A}\cos \mathrm{B} + \cos \mathrm{A}\sin \mathrm{B}$

(b) $\sin(\mathrm{A}-\mathrm{B}) = \sin \mathrm{A}\cos \mathrm{B} - \cos \mathrm{A}\sin \mathrm{B}$

(c) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

(d) $\cos(\mathrm{A}-\mathrm{B}) = \cos \mathrm{A}\cos \mathrm{B} + \sin \mathrm{A}\sin \mathrm{B}$

(e) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(f) $\tan(\mathrm{A}-\mathrm{B}) = \dfrac{\tan \mathrm{A} - \tan \mathrm{B}}{1 + \tan \mathrm{A}\tan \mathrm{B}}$

(g) $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

(f) $\cot(\mathrm{A}-\mathrm{B}) = \dfrac{\cot \mathrm{A}\cot \mathrm{B} + 1}{\cot \mathrm{B} - \cot \mathrm{A}}$

(h) $\sin^2 \mathrm{A} - \sin^2 \mathrm{B} = \cos^2 \mathrm{B} - \cos^2 \mathrm{A} = \sin(\mathrm{A}+\mathrm{B})\cdot\sin(\mathrm{A}-\mathrm{B})$

(i) $\cos^2 \mathrm{A} - \sin^2 \mathrm{B} = \cos^2 \mathrm{B} - \sin^2 \mathrm{A} = \cos(\mathrm{A}+\mathrm{B})\cdot\cos(\mathrm{A}-\mathrm{B})$

(j) $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A}$

Multiple Angles and Half Angles

(a) $\sin 2\,\mathrm{A} = 2\sin \mathrm{A}\cos \mathrm{A}; \quad \sin\theta = 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

(b) $\cos 2\,\mathrm{A} = \cos^2 \mathrm{A} - \sin^2 \mathrm{A} = 2\cos^2 \mathrm{A} - 1 = 1 - 2\sin^2 \mathrm{A}$

$2\cos^2\dfrac{\theta}{2} = 1 + \cos\theta, 2\sin^2\dfrac{\theta}{2} = 1 - \cos\theta$

(c) $\tan 2\,\mathrm{A} = \dfrac{2\tan \mathrm{A}}{1 - \tan^2 \mathrm{A}}; \tan\theta = \dfrac{2\tan\dfrac{\theta}{2}}{1 - \tan^2\dfrac{\theta}{2}}$

(d) $\sin 2\,\mathrm{A} = \dfrac{2\tan \mathrm{A}}{1 - \tan^2 \mathrm{A}}; \cos 2\,\mathrm{A} = \dfrac{1 - \tan^2 \mathrm{A}}{1 + \tan^2 \mathrm{A}}$

(e) $\sin 3\,\mathrm{A} = 3\sin \mathrm{A} - 4\sin^3 \mathrm{A}$

(f) $\cos 3\,\mathrm{A} = 4\cos^3 \mathrm{A} - 3\cos \mathrm{A}$

Transformation of Products into Sum or Difference of Sines & Cosines

(a) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(b) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(c) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(d) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Factorisation of the Sum or Difference of Two Sines or Cosines

(a) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

(b) $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

(c) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(d) $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$