**Example 9** Find the value of cos (–1710°).

**Solution** We know that values of cos $x$ repeats after an interval of $2\pi$ or 360°. Therefore, cos (–1710°) = cos (–1710° + 5 × 360°)

$$= \cos(-1710° + 1800°) = \cos 90° = 0.$$

## EXERCISE 3.2

Find the values of other five trigonometric functions in Exercises 1 to 5.

1. $\cos x = -\frac{1}{2}$, $x$ lies in third quadrant.
2. $\sin x = \frac{3}{5}$, $x$ lies in second quadrant.
3. $\cot x = \frac{3}{4}$, $x$ lies in third quadrant.
4. $\sec x = \frac{13}{5}$, $x$ lies in fourth quadrant.
5. $\tan x = -\frac{5}{12}$, $x$ lies in second quadrant.

Find the values of the trigonometric functions in Exercises 6 to 10.

6. sin 765°
7. cosec (– 1410°)
8. $\tan \frac{19\pi}{3}$
9. $\sin (-\frac{11\pi}{3})$
10. $\cot (-\frac{15\pi}{4})$

## 3.4 Trigonometric Functions of Sum and Difference of Two Angles

In this Section, we shall derive expressions for trigonometric functions of the sum and difference of two numbers (angles) and related expressions. The basic results in this connection are called *trigonometric identities*. We have seen that

1. **$\sin(-x) = -\sin x$**
2. **$\cos(-x) = \cos x$**

We shall now prove some more results:

**3.** $\cos(x+y) = \cos x \cos y - \sin x \sin y$

Consider the unit circle with centre at the origin. Let $x$ be the angle $P_4OP_1$ and $y$ be the angle $P_1OP_2$. Then $(x + y)$ is the angle $P_4OP_2$. Also let $(-y)$ be the angle $P_4OP_3$. Therefore, $P_1$, $P_2$, $P_3$ and $P_4$ will have the coordinates $P_1(\cos x, \sin x)$, $P_2[\cos(x+y), \sin(x+y)]$, $P_3[\cos(-y), \sin(-y)]$ and $P_4(1, 0)$ (Fig 3.14).

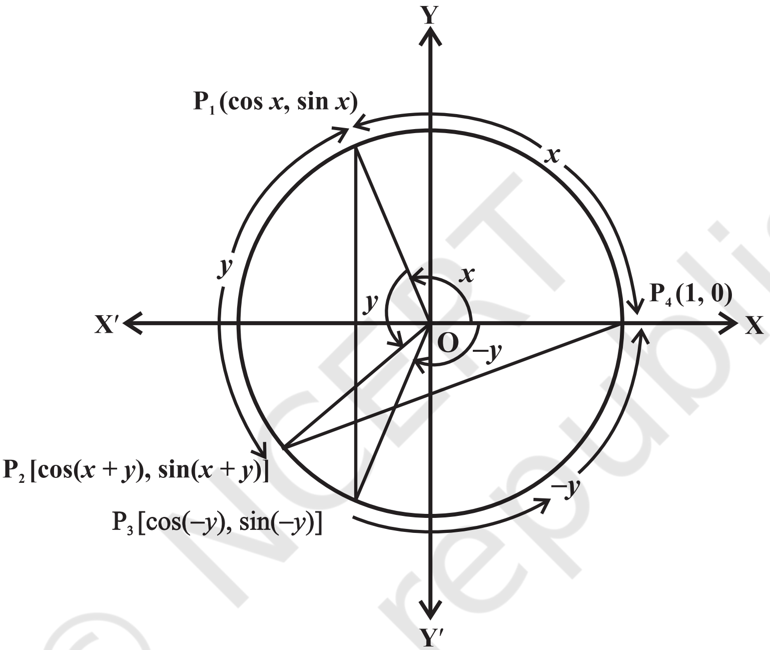


**Fig 3.14**

Consider the triangles $P_1OP_3$ and $P_2OP_4$. They are congruent (Why?). Therefore, $P_1P_3$ and $P_2P_4$ are equal. By using distance formula, we get

$$P_1P_3^2 = [\cos x - \cos(-y)]^2 + [\sin x - \sin(-y)]^2$$
$$= (\cos x - \cos y)^2 + (\sin x + \sin y)^2$$
$$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y + 2\sin x \sin y$$
$$= 2 - 2(\cos x \cos y - \sin x \sin y) \qquad \text{(Why?)}$$

Also, $$P_2P_4^2 = [1 - \cos(x+y)]^2 + [0 - \sin(x+y)]^2$$
$$= 1 - 2\cos(x+y) + \cos^2(x+y) + \sin^2(x+y)$$
$$= 2 - 2\cos(x+y)$$

Since $P_1P_3 = P_2P_4$, we have $P_1P_3^2 = P_2P_4^2$.

Therefore, $2 - 2(\cos x \cos y - \sin x \sin y) = 2 - 2\cos(x + y)$.

Hence $\cos(x + y) = \cos x \cos y - \sin x \sin y$

**4. $\cos(x - y) = \cos x \cos y + \sin x \sin y$**

Replacing $y$ by $-y$ in identity 3, we get

$$\cos(x + (-y)) = \cos x \cos(-y) - \sin x \sin(-y)$$

or $$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

**5. $\cos\left(\frac{\pi}{2} - x\right) = \sin x$**

If we replace $x$ by $\frac{\pi}{2}$ and $y$ by $x$ in Identity (4), we get

$$\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x = \sin x.$$

**6. $\sin\left(\frac{\pi}{2} - x\right) = \cos x$**

Using the Identity 5, we have

$$\sin\left(\frac{\pi}{2} - x\right) = \cos\left[\frac{\pi}{2} - \left(\frac{\pi}{2} - x\right)\right] = \cos x.$$

**7. $\sin(x + y) = \sin x \cos y + \cos x \sin y$**

We know that

$$\sin(x + y) = \cos\left(\frac{\pi}{2} - (x + y)\right) = \cos\left(\left(\frac{\pi}{2} - x\right) - y\right)$$

$$= \cos\left(\frac{\pi}{2} - x\right)\cos y + \sin\left(\frac{\pi}{2} - x\right)\sin y$$

$$= \sin x \cos y + \cos x \sin y$$

**8. $\sin(x - y) = \sin x \cos y - \cos x \sin y$**

If we replace $y$ by $-y$, in the Identity 7, we get the result.

**9.** By taking suitable values of $x$ and $y$ in the identities 3, 4, 7 and 8, we get the following results:

$$\cos\left(\frac{\pi}{2} + x\right) = -\sin x \qquad \sin\left(\frac{\pi}{2} + x\right) = \cos x$$

$$\cos(\pi - x) = -\cos x \qquad \sin(\pi - x) = \sin x$$

$$\cos(\pi + x) = -\cos x \qquad \sin(\pi + x) = -\sin x$$
$$\cos(2\pi - x) = \cos x \qquad \sin(2\pi - x) = -\sin x$$

Similar results for tan $x$, cot $x$, sec $x$ and cosec $x$ can be obtained from the results of sin $x$ and cos $x$.

**10. If none of the angles $x$, $y$ and $(x + y)$ is an odd multiple of $\frac{\pi}{2}$, then**

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Since none of the $x$, $y$ and $(x + y)$ is an odd multiple of $\frac{\pi}{2}$, it follows that cos $x$, cos $y$ and cos $(x + y)$ are non-zero. Now

$$\tan(x + y) = \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}.$$

Dividing numerator and denominator by cos $x$ cos $y$, we have

$$\tan(x + y) = \frac{\frac{\sin x \cos y}{\cos x \cos y} + \frac{\cos x \sin y}{\cos x \cos y}}{\frac{\cos x \cos y}{\cos x \cos y} - \frac{\sin x \sin y}{\cos x \cos y}}$$

$$= \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

**11.** $$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

If we replace $y$ by $-y$ in Identity 10, we get

$$\tan(x - y) = \tan[x + (-y)]$$

$$= \frac{\tan x + \tan(-y)}{1 - \tan x \tan(-y)} = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

**12. If none of the angles $x$, $y$ and $(x + y)$ is a multiple of $\pi$, then**

$$\cot(x + y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$$

Since, none of the $x$, $y$ and $(x + y)$ is multiple of $\pi$, we find that $\sin x \sin y$ and $\sin(x + y)$ are non-zero. Now,

$$\cot(x+y) = \frac{\cos(x+y)}{\sin(x+y)} = \frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y + \cos x \sin y}$$

Dividing numerator and denominator by $\sin x \sin y$, we have

$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$$

**13.** $\boldsymbol{\cot(x-y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}}$ if none of angles $x$, $y$ and $x–y$ is a multiple of $\pi$

If we replace $y$ by $–y$ in identity 12, we get the result

**14.** $\boldsymbol{\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \dfrac{1-\tan^2 x}{1+\tan^2 x}}$

We know that

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

Replacing $y$ by $x$, we get

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

Again, $\cos 2x = \cos^2 x - \sin^2 x$
$$= 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x.$$

We have $\cos 2x = \cos^2 x - \sin^2 x = \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$

Dividing numerator and denominator by $\cos^2 x$, we get

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}, \quad x \neq n\pi + \frac{\pi}{2}, \text{ where } n \text{ is an integer}$$

**15.** $\boldsymbol{\sin 2x = 2\sin x \cos x = \dfrac{2\tan x}{1+\tan^2 x}}$ $\quad x \neq n\pi + \dfrac{\pi}{2}$, where n is an integer

We have

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

Replacing $y$ by $x$, we get $\sin 2x = 2\sin x \cos x$.

Again $\sin 2x = \dfrac{2\sin x \cos x}{\cos^2 x + \sin^2 x}$

Dividing each term by $\cos^2 x$, we get

$$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$$

**16.** $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$ if $2x \neq n\pi + \dfrac{\pi}{2}$, where n is an integer

We know that

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Replacing $y$ by $x$, we get $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

**17.** $\sin 3x = 3\sin x - 4\sin^3 x$

We have,

$$\begin{aligned}\sin 3x &= \sin(2x+x)\\ &= \sin 2x \cos x + \cos 2x \sin x\\ &= 2\sin x \cos x \cos x + (1-2\sin^2 x)\sin x\\ &= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x\\ &= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x\\ &= 3\sin x - 4\sin^3 x\end{aligned}$$

**18.** $\cos 3x = 4\cos^3 x - 3\cos x$

We have,

$$\begin{aligned}\cos 3x &= \cos(2x+x)\\ &= \cos 2x \cos x - \sin 2x \sin x\\ &= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x\\ &= (2\cos^2 x - 1)\cos x - 2\cos x(1-\cos^2 x)\\ &= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x\\ &= 4\cos^3 x - 3\cos x.\end{aligned}$$

**19.** $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$ if $3x \neq n\pi + \dfrac{\pi}{2}$, where n is an integer

We have $\tan 3x = \tan(2x+x)$

$$= \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x} = \frac{\dfrac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \dfrac{2\tan x \,.\, \tan x}{1-\tan^2 x}}$$

$$= \frac{2\tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2\tan^2 x} = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

**20.** **(i)** $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

**(ii)** $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

**(iii)** $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

**(iv)** $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

We know that

$\cos(x + y) = \cos x \cos y - \sin x \sin y$ ... (1)

and $\cos(x - y) = \cos x \cos y + \sin x \sin y$ ... (2)

Adding and subtracting (1) and (2), we get

$\cos(x + y) + \cos(x - y) = 2\cos x \cos y$ ... (3)

and $\cos(x + y) - \cos(x - y) = -2\sin x \sin y$ ... (4)

Further $\sin(x + y) = \sin x \cos y + \cos x \sin y$ ... (5)

and $\sin(x - y) = \sin x \cos y - \cos x \sin y$ ... (6)

Adding and subtracting (5) and (6), we get

$\sin(x + y) + \sin(x - y) = 2\sin x \cos y$ ... (7)

$\sin(x + y) - \sin(x - y) = 2\cos x \sin y$ ... (8)

Let $x + y = \theta$ and $x - y = \phi$. Therefore

$$x = \left(\frac{\theta+\phi}{2}\right) \text{and } y = \left(\frac{\theta-\phi}{2}\right)$$

Substituting the values of $x$ and $y$ in (3), (4), (7) and (8), we get

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$

$$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$

Since θ and ϕ can take any real values, we can replace θ by $x$ and ϕ by $y$. Thus, we get

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}\ ;\ \cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2},$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}\ ;\ \sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}.$$

***Remark*** As a part of identities given in 20, we can prove the following results:

**21.** **(i)** $2\cos x\cos y = \cos(x+y) + \cos(x-y)$

**(ii)** $-2\sin x\sin y = \cos(x+y) - \cos(x-y)$

**(iii)** $2\sin x\cos y = \sin(x+y) + \sin(x-y)$

**(iv)** $2\cos x\sin y = \sin(x+y) - \sin(x-y)$.

**Example 10** Prove that

$$3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4} = 1$$

**Solution** We have

$$\text{L.H.S.} = 3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}$$

$$= 3\times\frac{1}{2}\times 2 - 4\sin\left(\pi - \frac{\pi}{6}\right)\times 1 = 3 - 4\sin\frac{\pi}{6}$$

$$= 3 - 4\times\frac{1}{2} = 1 = \text{R.H.S.}$$

**Example 11** Find the value of sin 15°.

**Solution** We have

$$\sin 15^\circ = \sin(45^\circ - 30^\circ)$$

$$= \sin 45^\circ\cos 30^\circ - \cos 45^\circ\sin 30^\circ$$

$$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}.$$

**Example 12** Find the value of $\tan\frac{13\pi}{12}$.

**Solution** We have

$$\tan\frac{13\pi}{12} = \tan\left(\pi+\frac{\pi}{12}\right) = \tan\frac{\pi}{12} = \tan\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$$

$$= \frac{\tan\frac{\pi}{4}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{4}\tan\frac{\pi}{6}} = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$$

**Example 13** Prove that

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x+\tan y}{\tan x-\tan y}.$$

**Solution** We have

$$\text{L.H.S.} \quad = \frac{\sin(x+y)}{\sin(x-y)} = \frac{\sin x\cos y+\cos x\sin y}{\sin x\cos y-\cos x\sin y}$$

Dividing the numerator and denominator by cos $x$ cos $y$, we get

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x+\tan y}{\tan x-\tan y}.$$

**Example 14** Show that

$$\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$$

**Solution** We know that $3x = 2x + x$

Therefore, $\tan 3x = \tan(2x + x)$

or $$\tan 3x = \frac{\tan 2x+\tan x}{1-\tan 2x\tan x}$$

or $\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$

or $\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$

or $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x.$

**Example 15** Prove that

$$\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)=\sqrt{2}\cos x$$

**Solution** Using the Identity 20(i), we have

$$\text{L.H.S.} = \cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)$$

$$= 2\cos\left(\frac{\frac{\pi}{4}+x+\frac{\pi}{4}-x}{2}\right)\cos\left(\frac{\frac{\pi}{4}+x-(\frac{\pi}{4}-x)}{2}\right)$$

$$= 2\cos\frac{\pi}{4}\cos x = 2\times\frac{1}{\sqrt{2}}\cos x = \sqrt{2}\cos x = \text{R.H.S.}$$

**Example 16** Prove that $\dfrac{\cos 7x+\cos 5x}{\sin 7x-\sin 5x}=\cot x$

**Solution** Using the Identities 20 (i) and 20 (iv), we get

$$\text{L.H.S.} = \frac{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}}{2\cos\frac{7x+5x}{2}\sin\frac{7x-5x}{2}} = \frac{\cos x}{\sin x} = \cot x = \text{R.H.S.}$$

**Example 17** Prove that $\dfrac{\sin 5x-2\sin 3x+\sin x}{\cos 5x-\cos x}=\tan x$

**Solution** We have

$$\text{L.H.S.} = \frac{\sin 5x-2\sin 3x+\sin x}{\cos 5x-\cos x} = \frac{\sin 5x+\sin x-2\sin 3x}{\cos 5x-\cos x}$$

$$= \frac{2\sin 3x\cos 2x-2\sin 3x}{-2\sin 3x\sin 2x} = -\frac{\sin 3x(\cos 2x-1)}{\sin 3x\sin 2x}$$

$$= \frac{1-\cos 2x}{\sin 2x} = \frac{2\sin^2 x}{2\sin x\cos x} = \tan x = \text{R.H.S.}$$

## EXERCISE 3.3

Prove that:

**1.** $\sin^2 \dfrac{\pi}{6} + \cos^2 \dfrac{\pi}{3} - \tan^2 \dfrac{\pi}{4} = -\dfrac{1}{2}$

**2.** $2\sin^2 \dfrac{\pi}{6} + \operatorname{cosec}^2 \dfrac{7\pi}{6} \cos^2 \dfrac{\pi}{3} = \dfrac{3}{2}$

**3.** $\cot^2 \dfrac{\pi}{6} + \operatorname{cosec} \dfrac{5\pi}{6} + 3\tan^2 \dfrac{\pi}{6} = 6$

**4.** $2\sin^2 \dfrac{3\pi}{4} + 2\cos^2 \dfrac{\pi}{4} + 2\sec^2 \dfrac{\pi}{3} = 10$

**5.** Find the value of:

(i) $\sin 75°$ (ii) $\tan 15°$

Prove the following:

**6.** $\cos\left(\dfrac{\pi}{4} - x\right)\cos\left(\dfrac{\pi}{4} - y\right) - \sin\left(\dfrac{\pi}{4} - x\right)\sin\left(\dfrac{\pi}{4} - y\right) = \sin(x + y)$

**7.** $\dfrac{\tan\left(\dfrac{\pi}{4} + x\right)}{\tan\left(\dfrac{\pi}{4} - x\right)} = \left(\dfrac{1 + \tan x}{1 - \tan x}\right)^2$

**8.** $\dfrac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\dfrac{\pi}{2} + x\right)} = \cot^2 x$

**9.** $\cos\left(\dfrac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\dfrac{3\pi}{2} - x\right) + \cot(2\pi + x)\right] = 1$

**10.** $\sin(n + 1)x \sin(n + 2)x + \cos(n + 1)x \cos(n + 2)x = \cos x$

**11.** $\cos\left(\dfrac{3\pi}{4} + x\right) - \cos\left(\dfrac{3\pi}{4} - x\right) = -\sqrt{2}\sin x$

**12.** $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

**13.** $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

**14.** $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

**15.** $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

**16.** $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\dfrac{\sin 2x}{\cos 10x}$

**17.** $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

**18.** $\dfrac{\sin x - \sin y}{\cos x + \cos y} = \tan \dfrac{x - y}{2}$

**19.** $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

**20.** $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

**21.** $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

**22.** $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

**23.** $\tan 4x = \dfrac{4\tan x\,(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

**24.** $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

**25.** $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

## 3.5 Trigonometric Equations

Equations involving trigonometric functions of a variable are called *trigonometric equations*. In this Section, we shall find the solutions of such equations. We have already learnt that the values of $\sin x$ and $\cos x$ repeat after an interval of $2\pi$ and the values of $\tan x$ repeat after an interval of $\pi$. The solutions of a trigonometric equation for which $0 \le x < 2\pi$ are called *principal solutions*. The expression involving integer '$n$' which gives all solutions of a trigonometric equation is called the *general solution*. We shall use '**Z**' to denote the set of integers.

The following examples will be helpful in solving trigonometric equations:

**Example 18** Find the principal solutions of the equation $\sin x = \dfrac{\sqrt{3}}{2}$.

**Solution** We know that, $\sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$ and $\sin\dfrac{2\pi}{3} = \sin\left(\pi - \dfrac{\pi}{3}\right) = \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$.

Therefore, principal solutions are $x = \dfrac{\pi}{3}$ and $\dfrac{2\pi}{3}$.

**Example 19** Find the principal solutions of the equation $\tan x = -\dfrac{1}{\sqrt{3}}$.

**Solution** We know that, $\tan\dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}}$. Thus, $\tan\left(\pi - \dfrac{\pi}{6}\right) = -\tan\dfrac{\pi}{6} = -\dfrac{1}{\sqrt{3}}$

and $\tan\left(2\pi - \dfrac{\pi}{6}\right) = -\tan\dfrac{\pi}{6} = -\dfrac{1}{\sqrt{3}}$

Thus $\tan\dfrac{5\pi}{6} = \tan\dfrac{11\pi}{6} = -\dfrac{1}{\sqrt{3}}$.

Therefore, principal solutions are $\dfrac{5\pi}{6}$ and $\dfrac{11\pi}{6}$.

We will now find the general solutions of trigonometric equations. We have already