

Trigonometry Functions - Class XI

Past Year JEE Questions

Questions

Question: 01

The value of $\cos \frac{\pi}{2} \cdot \cos \frac{\pi}{2^2} \cdot \dots \cdot \cos \frac{\pi}{2^8} \sin \frac{\pi}{2^9}$ is -

- A. $\frac{1}{256}$
- B. $\frac{1}{2}$
- C. $\frac{1}{1024}$
- D. $\frac{1}{512}$

Solutions

Solution: 01

Explanation

Given $\cos \frac{\pi}{2} \cdot \cos \frac{\pi}{2^2} \cdot \dots \cdot \cos \frac{\pi}{2^8} \sin \frac{\pi}{2^9}$

Let $\frac{\pi}{2^9} = \theta$

$$\therefore \frac{\pi}{2} = 2\theta$$

$$\frac{\pi}{2^2} = 2^2\theta$$

$$\frac{\pi}{2^3} = 2^3\theta$$

.

.

$$\frac{\pi}{2^8} = 2^8\theta$$

So given term becomes,

$$\cos 2^8\theta \cdot \cos 2^7\theta \cdot \dots \cdot \cos \theta \cdot \sin \frac{\pi}{2^9}$$

$$= (\cos \theta \cdot \cos 2\theta \cdot \dots \cdot \cos 2^8\theta) \sin \frac{\pi}{2^9}$$

$$= \frac{\sin 2^9\theta}{2^9 \sin \theta} \sin \frac{\pi}{2^9}$$

$$= \frac{\sin 2^9\left(\frac{\pi}{2^9}\right)}{2^9 \sin \frac{\pi}{2^9}} \sin \frac{\pi}{2^9}$$

$$= \frac{\sin(\frac{\pi}{2})}{2^9}$$

$$= \frac{1}{2^9} = \frac{1}{512}$$

Note :

$$(\cos\theta.\cos2\theta.\dots\dots\cos2^{n-1}\theta)=\frac{\sin2^n\theta}{2^n\sin\theta}$$