23. If a cos2θ + b sin 2θ = c has α and β as its roots, then prove that tan α + tan β = 2b/(a + c)

[Hint: Use the identities cos 2θ = ((1 – tan ^{2} θ)/(1 + tan ^{2} θ) and sin 2θ = 2tan θ/(1 + tan ^{2} θ)]

Solution:

According to the question,

a cos2θ + b sin 2θ = c

α and β are the roots of the equation.

Using the formula of multiple angles,

We know that,

$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ and $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$\therefore a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) - c = 0$

⇒ a(1 – tan ^{2} θ) + 2b tan θ – c(1 + tan ^{2} θ) = 0

⇒ (-c – a)tan ^{2} θ + 2b tan θ – c + a = 0 …(i)

We know that,

The sum of roots of a quadratic equation, ax ^{2} + bx + c = 0 is given by (-b/a)

Therefore,

tan α + tan β = –2b/–(c + a) = 2b/(c + a)

Hence, tan α + tan β = 2b/(c + a)