22. Find the value of the expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$$

Solution:

According to the question,

Let, $y = 3[\sin^4(3\pi/2 - \alpha) + \sin^4(3\pi + \alpha)] - 2[\sin^6(\pi/2 + \alpha) + \sin^6(5\pi - \alpha)]$

We know that,

$\sin(3\pi/2 - \alpha) = -\cos\alpha$

$\sin(3\pi + \alpha) = -\sin\alpha$

$\sin(\pi/2 + \alpha) = \cos\alpha$

$\sin(5\pi - \alpha) = \sin\alpha$

Therefore,

$y = 3[(-\cos\alpha)^4 + (-\sin\alpha)^4] - 2[\cos^6\alpha + \sin^6\alpha]$

$\Rightarrow y = 3[\cos^4\alpha + \sin^4\alpha] - 2[\sin^6\alpha + \cos^6\alpha]$

$\Rightarrow y = 3[(\sin^2\alpha + \cos^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha] - 2[(\sin^2\alpha)^3 + (\cos^2\alpha)^3]$

Since, we know that,

$\sin^2\alpha + \cos^2\alpha = 1$

Also, we know that,

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$\Rightarrow y = 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[(\sin^2\alpha + \cos^2\alpha)(\cos^4\alpha + \sin^4\alpha - \sin^2\alpha\cos^2\alpha)]$

$\Rightarrow y = 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[\cos^4\alpha + \sin^4\alpha - \sin^2\alpha\cos^2\alpha]$

$\Rightarrow y = 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[(\sin^2\alpha + \cos^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha - \sin^2\alpha\cos^2\alpha]$

$\Rightarrow y = 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[1 - 3\sin^2\alpha\cos^2\alpha]$

$\Rightarrow y = 3 - 6\sin^2\alpha\cos^2\alpha - 2 + 6\sin^2\alpha\cos^2\alpha$

$\Rightarrow y = 1$