21. If cos (θ + ϕ) = m cos (θ − ϕ), then prove that tan θ = ((1 − m)/(1 + m)) cot ϕ
[Hint: Express cos (θ + ϕ)/ cos (θ − ϕ) = m/l and apply Componendo and Dividendo] Solution:

According to the question,

$\cos(\theta + \phi) = m\cos(\theta - \phi)$

$\because \cos(\theta + \phi) = m\cos(\theta - \phi)$

$\Rightarrow \frac{\cos(\theta-\phi)}{\cos(\theta+\phi)} = \frac{1}{m}$

Applying componendo – dividend, we get,

$\Rightarrow \frac{\cos(\theta-\phi)+\cos(\theta+\phi)}{\cos(\theta-\phi)-\cos(\theta+\phi)} = \frac{1+m}{1-m}$

From transformation formula, we know that,

$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$

$\cos(A-B) - \cos(A+B) = 2\sin A\sin B$

$\Rightarrow \frac{2\cos\theta\cos\phi}{2\sin\theta\sin\phi} = \frac{1+m}{1-m}$

Since, $(\cos\theta)/(\sin\theta) = \cot\theta$

$\Rightarrow \cot\theta\cot\phi = \frac{1+m}{1-m}$

$\Rightarrow \left(\frac{1-m}{1+m}\right)\cot\phi = \frac{1}{\cot\theta}$

$\Rightarrow \tan\theta = \left(\frac{1-m}{1+m}\right)\cot\phi$