**3. If m sin θ = n sin (θ + 2α), then prove that**

**tan (θ + α) cot α = (m + n)/(m – n)**

**[Hints: Express sin(θ + 2α) / sinθ = m/n and apply componendo and dividend]**

**Solution:**

According to the question,

m sin θ = n sin (θ + 2α)

To prove:

tan (θ + α)cot α =(m + n)/(m – n)

Proof:

m sin θ = n sin (θ + 2α)

⇒ sin(θ + 2α) / sinθ = m/n

Applying componendo-dividendo rule, we have,

$$\Rightarrow \frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta} = \frac{m+n}{m-n}$$

By transformation formula of T-ratios,

We know that,

sin A + sin B = 2 sin ((A+B)/2) cos ((A – B)/2)

And,

sin A – sin B = 2 cos ((A+B)/2) sin ((A – B)/2)

On applying the formula, we get,

$$\frac{2\sin\left(\frac{2\theta+2\alpha}{2}\right)\cos\left(\frac{\theta+2\alpha-\theta}{2}\right)}{2\cos\left(\frac{2\theta+2\alpha}{2}\right)\sin\left(\frac{\theta+2\alpha-\theta}{2}\right)} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{\sin(\theta+\alpha)\cos(\alpha)}{\cos(\theta+\alpha)\sin(\alpha)} = \frac{m+n}{m-n}$$

$\{\because \tan x = (\sin x)/(\cos x)\}$

$$\Rightarrow \tan(\theta+\alpha)\cot\alpha = \frac{m+n}{m-n}$$

Therefore, tan (θ + α) cot α = (m + n)/(m – n)

Hence Proved