त्रिकोणमिति

त्रिकोणमितिय फलन $f(\theta) = \sin\theta, \cos\theta, \operatorname{cosec}\theta, \sec\theta, \tan\theta, \cot\theta$

तीन पद्धति :- षाष्टिक पद्धति / ब्रिटिश पद्धति – डिग्री, मिनट, सेकण्ड
शातिक पद्धति – ग्रेड, कला, विकला
वृत्तीय पद्धति – रेडियन

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

1 समकोण = $90°$

1 समकोण = 100^g

1 समकोण = $\left(\frac{\pi}{2}\right)^c$

π रेडियन $= 180°$, 1 रेडियन $= \left(\frac{180}{\pi}\right)° =$ (एक समकोण का $\frac{2}{\pi}$)

$$\boxed{\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}}$$

1 रेडियन $= 57°16'22''$ (लगभग) / $57°17'45''$ (लगभग)

Q ~~sin 1~~ निम्न में से कौनसा कथन सत्य है?

$\sin 1 > \sin 1°$ —

$\sin 1 < \sin 1°$

$\sin 1 = \sin 1°$

None of thes

$\sec\theta + \tan\theta = x$ हो तब $\sec\theta - \tan\theta = \frac{1}{x}$

परन्तु $\tan\theta + \sec\theta = x$ हो तब $\tan\theta - \sec\theta = -\frac{1}{x}$

$\operatorname{cosec}\theta + \cot\theta = x$ हो तब $\operatorname{cosec}\theta - \cot\theta = \frac{1}{x}$

$\cot\theta + \operatorname{cosec}\theta = x$ हो तब $\cot\theta - \operatorname{cosec}\theta = -1/x$

Q. $\sin(10°6'30'') = a$ तब

$\cos(79°53'28'') + \tan(10°6'30'')$ का मान होगा

त्रिज्या

i) $\frac{a(1+\sqrt{1-a^2})}{\sqrt{1-a^2}}$ ✓ (ii) $\frac{a\sqrt{1-a^2}+1}{\sqrt{1-a^2}}$

(iii) $\frac{1+\sqrt{1-a^2}}{\sqrt{1-a^2}}$ (v) $\frac{\sqrt{1-a^2}+a}{\sqrt{1-a^2}}$

<u>Ans</u> $10^\circ 6' 32'' = \alpha$ $\quad 79^\circ 53' 28'' = \beta$

$\alpha + \beta = 90$

$\cos\beta + \tan\alpha = \cos(90-\alpha) + \tan\alpha$

$= \sin\alpha + \tan\alpha$

$= a + \frac{a}{\sqrt{1-a^2}}$

$= a\left(\frac{\sqrt{1-a^2}+1}{\sqrt{1-a^2}}\right)$

★ $\sin(60-\theta)\ \sin\theta\ \sin(60+\theta) = \frac{1}{4}\sin 3\theta$

$\cos(60-\theta)\ \cos\theta\ \cos(60+\theta) = \frac{1}{4}\cos 3\theta$

$\tan(60-\theta)\ \tan\theta\ \tan(60+\theta) = \tan 3\theta$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$

$\cot(A+B) = \frac{\cot A\cot B - 1}{\cot A + \cot B}$

$\cot(A-B) = \frac{\cot A\cot B + 1}{\cot B - \cot A}$

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$-2\sin A\sin B = \cos(A+B) - \cos(A-B)$

$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$

$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$= \cos^2 A - \sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$

Scanned with CamScanner

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - \cancel{3} \; 3\cos A$

$\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

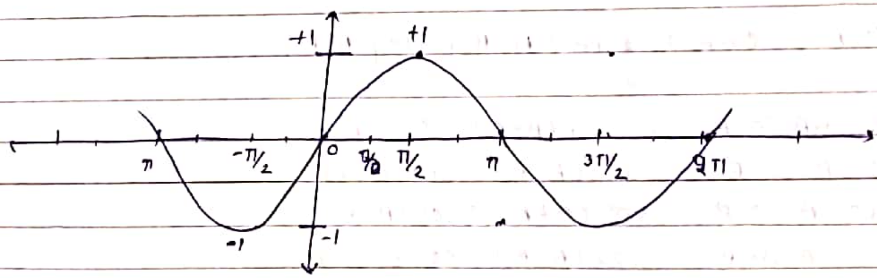
Q. $\cos(2820)$

$\cos(90 \times 31 + 30) = \cos(90 \times 31 + 30)$

$= +\sin(30^\circ) = \dfrac{+1}{2}$ Ans

$f(\theta) = \sin\theta$ Domain $= R$

Range $= [-1, 1]$



Q x के पूर्णांक मानों की संख्या जिसके लिए $\sin\theta = \dfrac{4x-3}{9}$ है तथा θ का पूर्णांक मान $0 \le \theta \le 90^\circ$ की Range

Ans $\sin\theta = \dfrac{4x-3}{9}$ $[0 \le \theta \le 90^\circ]$

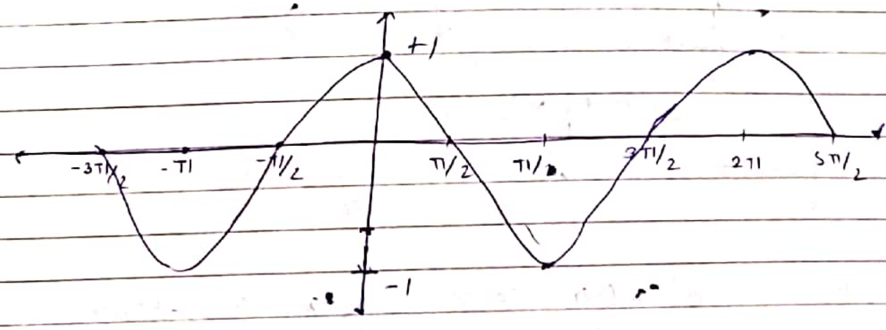
$0 \le \sin\theta \le 1 \Rightarrow 0 \le \dfrac{4x-3}{9} \le 1$

$0 \le 4x-3 \le 9 \Rightarrow 3 \le 4x \le 12 \Rightarrow \dfrac{3}{4} \le x \le 3$

$-1 \le \sin\theta \le 1 \Rightarrow 0 \le \sin^2\theta \le 1$

$-1 \le \cos\theta \le 1 \Rightarrow 0 \le \cos^2\theta \le 1$

$f(\theta) = \cos\theta$ Domain $= R$ Range $= [-1, 1]$



$-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$

$0 \le \sin^2\theta \le 1$ for $\theta \in R$

$0 \le \cos^2\theta \le 1$ for $\theta \in R$

Example $3\sin^2\theta + 4\cos^2\theta$

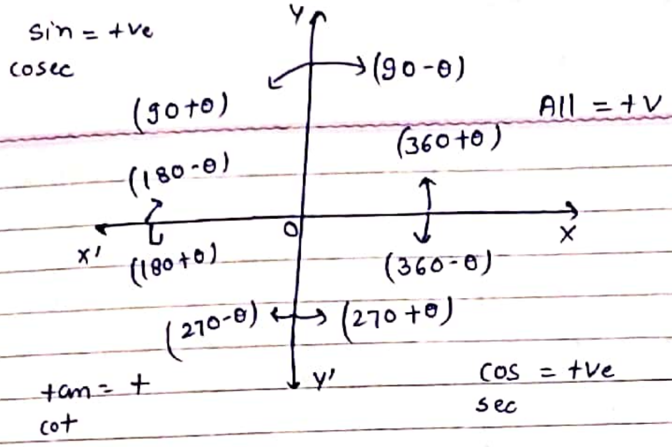
$0 \le 3\sin^2\theta + 4\cos^2\theta \le 1$

$0 \le 3\sin^2\theta + 4(1 - \sin^2\theta) \le 1$

$0 \le 3\sin^2\theta + 4 - 4\sin^2\theta \le 1$

$0 \le 4 - \sin^2\theta \le 1 = \cancel{-4 \le -\sin^2\theta \le -3}$

$\boxed{3 \le -\sin^2\theta \le 4}$



$\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$	$\tan(-\theta) = -\tan\theta$
$\sin(90-\theta) = \cos\theta$	$\cos(90-\theta) = \sin\theta$	$\tan(90-\theta) = \cot\theta$
$\sin(360+\theta) = \sin\theta$	$\cos(360+\theta) = \cos\theta$	$\tan(360+\theta) = \tan\theta$
$\sin(90+\theta) = \cos\theta$	$\cos(90+\theta) = -\sin\theta$	$\tan(90+\theta) = -\cot\theta$
$\sin(180-\theta) = \sin\theta$	$\cos(180-\theta) = -\cos\theta$	$\tan(180-\theta) = -\tan\theta$
$\sin(180+\theta) = -\sin\theta$	$\cos(180+\theta) = -\cos\theta$	$\tan(180+\theta) = \tan\theta$
$\sin(270-\theta) = -\cos\theta$	$\cos(270-\theta) = -\sin\theta$	$\tan(270-\theta) = \cot\theta$
$\sin(270+\theta) = -\cos\theta$	$\cos(270+\theta) = \sin\theta$	$\tan(270+\theta) = -\cot\theta$
$\sin(360-\theta) = -\sin\theta$	$\cos(360-\theta) = \cos\theta$	$\tan(360-\theta) = -\tan\theta$

$\text{cosec}(-\theta) = -\text{cosec}\theta$	$\sec(-\theta) = \sec\theta$	$\cot(-\theta) = -\cot\theta$
$\text{cosec}(90-\theta) = \sec\theta$	$\sec(90-\theta) = \text{cosec}\theta$	$\cot(90-\theta) = \tan\theta$
$\text{cosec}(90+\theta) = \sec\theta$	$\sec(90+\theta) = -\text{cosec}\theta$	$\cot(90+\theta) = -\tan\theta$
$\text{cosec}(180-\theta) = \text{cosec}\theta$	$\sec(180-\theta) = -\sec\theta$	$\cot(180-\theta) = -\cot\theta$
$\text{cosec}(180+\theta) = -\text{cosec}\theta$	$\sec(180+\theta) = -\sec\theta$	$\cot(180+\theta) = \cot\theta$
$\text{cosec}(270-\theta) = -\sec\theta$	$\sec(270-\theta) = -\text{cosec}\theta$	$\cot(270-\theta) = \tan\theta$
$\text{cosec}(270+\theta) = -\sec\theta$	$\sec(270+\theta) = \text{cosec}\theta$	$\cot(270+\theta) = -\tan\theta$
$\text{cosec}(360-\theta) = -\text{cosec}\theta$	$\sec(360-\theta) = \sec\theta$	$\cot(360-\theta) = -\cot\theta$
$\text{cosec}(360+\theta) = \text{cosec}\theta$	$\sec(360+\theta) = \sec\theta$	$\cot(360+\theta) = \cot\theta$

Q $A = \sin^4\theta + \cos^2\theta$ का Max व Min

Ans $A = \sin^4\theta + 1 - \sin^2\theta$

$= \sin^4\theta - \sin^2\theta + \frac{1}{4} + 1 - \frac{1}{4}$

$A = \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4}$

$-1 \le \sin\theta \le 1 \qquad \theta \in R$

$0 \le \sin^2\theta \le 1$

$\Rightarrow -\frac{1}{2} \le \sin^2\theta - \frac{1}{2} \le \frac{1}{2}$

$0 \le \left(\sin^2\theta - \frac{1}{2}\right)^2 \le \frac{1}{4}$

$\frac{3}{4} \le \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4} \le 1$

$\frac{3}{4} \le A \le 1$

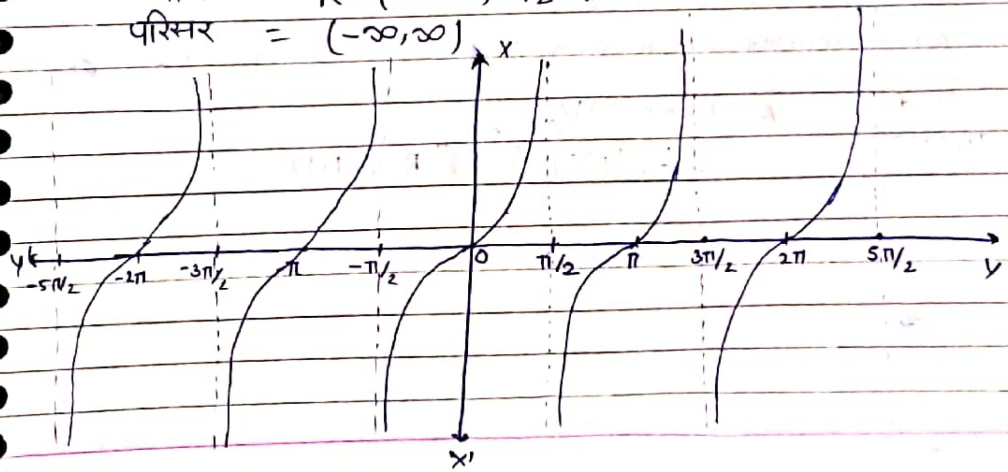
* $f(\theta) = \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = n\pi \ \forall \ n \in I$

$\tan\theta \rightarrow$ अपरिमित $\Rightarrow \cos\theta = 0 \Rightarrow \theta = (2n+1)\pi/2 \ \forall \ n \in I$

प्रान्त $= R - (2n+1)\pi/2 \ \forall \ n \in I$

परिसर $= (-\infty, \infty)$



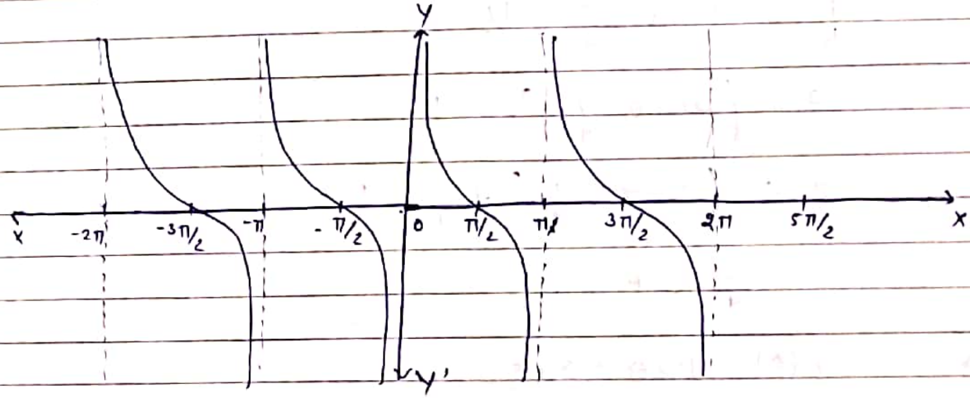
* $f(\theta) = \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\cot\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = (2n+1)\pi/2 \quad \forall\ n \in I$

$\cot\theta \longrightarrow$ अपरिमित $\Rightarrow \sin\theta = 0 \Rightarrow \theta = n\pi \quad \forall\ n \in I$

प्रांत $= R - n\pi \quad \forall\ n \in I$

परिसर $= (-\infty, \infty)$



* $f(\theta) = \sec\theta = \dfrac{1}{\cos\theta}$

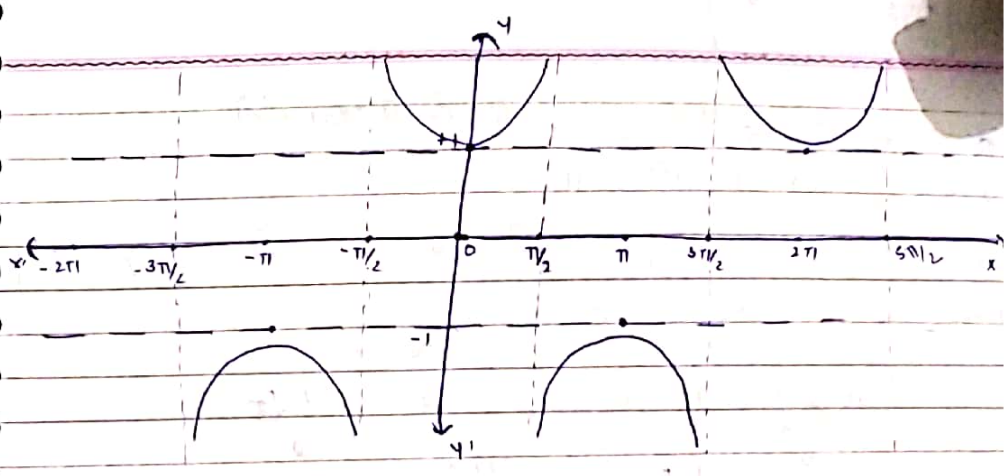
$\sec\theta = 1 \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0, 2\pi, 4\pi, 6\pi \ldots$

$\sec\theta = -1 \Rightarrow \cos\theta = -1 \Rightarrow \theta = \pi, 3\pi, 5\pi \ldots$

$\sec\theta \rightarrow$ अपरिमित $\Rightarrow \cos\theta = 0 \Rightarrow \theta = (2n+1)\pi/2 \quad \forall\ n \in I$

प्रांत $= R - (2n+1)\pi/2$

परिसर $= (-\infty, -1] \cup [1, \infty) \Rightarrow R - (-1, 1)$



* $f(\theta) = \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

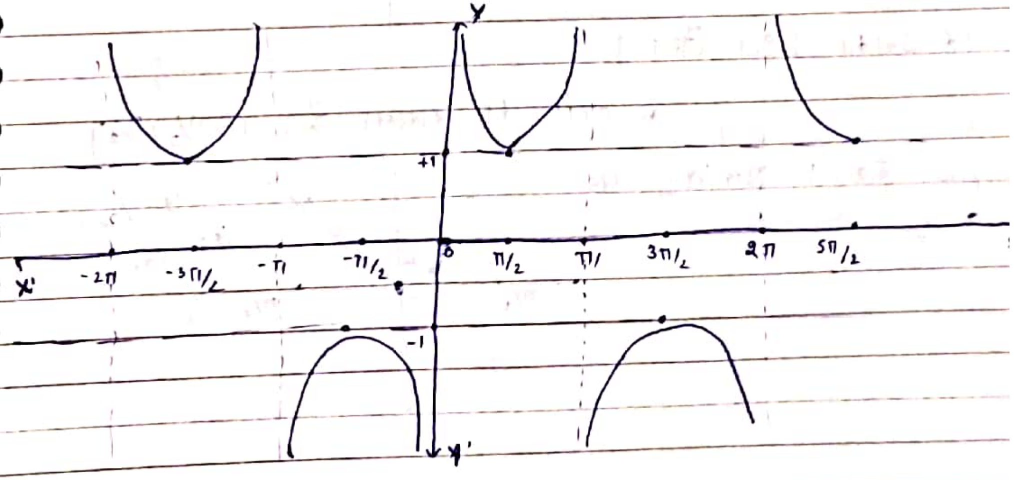
$\operatorname{cosec}\theta = 1 \Rightarrow \sin\theta = 1 \Rightarrow \theta = \pi/2, 5\pi/2$

$\operatorname{cosec}\theta = -1 \Rightarrow \sin\theta = -1 \Rightarrow \theta = 3\pi/2, 7\pi/2$

$\operatorname{cosec}\theta \longrightarrow$ अपरिमित $\Rightarrow \sin\theta = 0 \Rightarrow \theta = n\pi \quad \forall\ n \in I$

प्रान्त $- R - n\pi$

परिसर $- (-\infty, -1] \cup [1, \infty) = R - (-1, 1)$



$e = 2.7$ $\tan 15 = 2 - \sqrt{3}$ $0 \le |\sin x| \le 1$

$\boxed{\frac{1}{e} \le e^{\sin x} \le e}$

Q $e^{\sin x} - e^{-\sin x} - 6 = 0$ के कितने हल होंगे

Ans एक भी नहीं होगा

Q $e^{\sin x} - e^{-\sin x} - 1 = 0$ के कितने हल होंगे $[-\pi/2, \pi/2]$

Ans $e^{\sin x} = \alpha$, $e^{-\sin x} = \frac{1}{\alpha}$

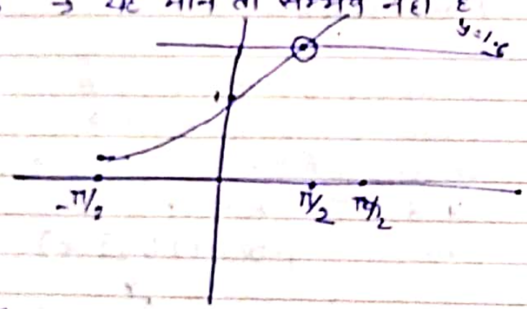
$\alpha - \frac{1}{\alpha} - 1 = 0 \Rightarrow \alpha = \frac{1 \pm \sqrt{5}}{2}$

$e^{\sin x} = \frac{1 \pm \sqrt{5}}{2} \Rightarrow e^{\sin x} = \frac{1+\sqrt{5}}{2}$

$e^{-\sin x} = \frac{1-\sqrt{5}}{2} \rightarrow$ यह मान तो सम्भव नहीं है

$e^{\sin x} = \frac{1+\sqrt{5}}{2}$

$y = \frac{1+\sqrt{5}}{2} = 1.6$

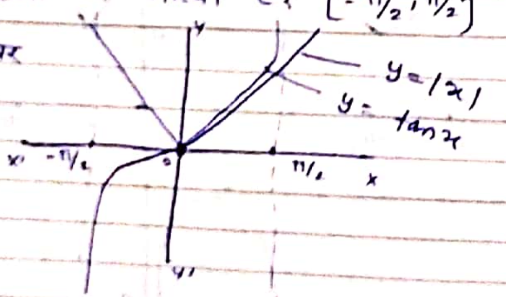


अतः इसका 1 हल होगा।

Q $\tan x = |x|$ के हलों की संख्या है? $[-\pi/2, \pi/2]$

Ans - केवल 1 = मूल बिन्दु पर

always $\tan x \ge x$



Q यदि A द्वितीय चतुर्थांश मे है और $3\tan A + 4 = 0$ है तो $2\cot A - 5\cos A + \sin A = ?$

Ans $\tan A = \frac{-4}{3}$ कर्ण $= \sqrt{16+9}$
$= \sqrt{25} = 5$

$2\left(-\frac{3}{4}\right) - 5\left(-\frac{3}{5}\right) + \frac{4}{5}$

$= -\frac{3}{2} + 3 + \frac{4}{5} = \frac{-15+30+8}{10} = \frac{15+8}{10} = \frac{23}{10}$

Q यदि θ III चतुर्थांश मे है और $\sin\theta = 4/5$ है तो $\cos\theta/2$ क्या होगा

(i) $\frac{1}{\sqrt{5}}$ (ii) $-\frac{1}{\sqrt{5}}$ (iii) $\sqrt{\frac{2}{5}}$ (iv) $-\sqrt{\frac{2}{5}}$

Ans $\sin\theta = \frac{4}{5}$ आधार $= \sqrt{25-16}$
$= \sqrt{9} = 3$

$\cos\theta = \frac{-3}{5}$

$\boxed{\begin{array}{l} \cos 2\theta = 2\cos^2\theta - 1 \\ \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \end{array}}$

$\pi < \theta < 3\pi/2 \Rightarrow \pi/2 < \theta/2 < 3\pi/4$

$\cos\frac{\theta}{2} = -\sqrt{\frac{1 - 3/5}{2}} = -\frac{1}{\sqrt{5}}$

$\sin(A+B) = \sin A\cos B + \cos A \sin B$ — ①

$B=-B$ $\quad \sin(A-B) = \sin A\cos B - \cos A \sin B$ — ②

समी ① + समी ②

$\boxed{\sin(A+B) + \sin(A-B) = 2\sin A\cos B}$

समी ① - समी ②

$\boxed{\sin(A+B) - \sin(A-B) = 2\cos A \sin B}$

$A+B = C \qquad A-B = D$

$A = \frac{C+D}{2} \qquad B = \frac{C-D}{2}$

$\boxed{\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}}$

$\boxed{\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}}$

समी ① X समी ②

$\sin(A+B)\sin(A-B) = \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$

$= \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$

$= \sin^2 A - \sin^2 A\sin^2 B - \sin^2 B + \sin^2 A\sin^2 B$

$\boxed{\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B}$

$\sin(A+B)\sin(A-B) = (1-\cos^2 A)\cos^2 B - \cos^2 A(1-\cos^2 B)$

$= \cos^2 B - \cos^2 A\cos^2 B - \cos^2 A + \cos^2 A\cos^2 B$

$\boxed{\sin(A+B)\sin(A-B) = \cos^2 B - \cos^2 A}$

$A=B$ रखने पर

$\sin 2A = \sin A\cos A + \cos A\sin A = 2\sin A\cos A$

$\boxed{\sin 2A = 2\sin A\cos A}$

$\boxed{\sin A = 2\sin\frac{A}{2}\cos A/_2}$

$\boxed{\sin 2A = \frac{2\sin A\cos A}{\sin^2 A + \cos^2 A} = \frac{2\tan A}{1+\tan^2 A}}$

समी ① में $B = 90+B$ रखने पर

$\sin[90+(A+B)] = \sin A\cos(90+B) + \cos A\sin(90+B)$

$\cos(A+B) = -\sin A\sin B + \cos A\cos B$

$\boxed{\cos(A+B) = \cos A\cos B - \sin A\sin B}$ — ③

$\boxed{\cos(A-B) = \cos A\cos B + \sin A\sin B}$ — ④

समी ③ + समी ④

$\boxed{\cos(A+B)+\cos(A-B) = 2\cos A\cos B}$

समी ③ - समी ④

$\boxed{\cos(A+B) - \cos(A-B) = -2\sin A\sin B}$

$A+B = C, \; A-B = D \Rightarrow A = \frac{C+D}{2}, \; B = \frac{C-D}{2}$

$\boxed{\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)}$

$\boxed{\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)}$

समी ③ X समी ④

$\cos(A+B)\cos(A-B) = \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$

$= \cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B$

$= \cos^2 A - \sin^2 B\cos^2 A - \sin^2 B + \cos^2 A\sin^2 B$

$\boxed{\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B}$

$\cos(A+B)\cos(A-B) = (1-\sin^2 A)\cos^2 B - \sin^2 A(1-\cos^2 B)$

$= \cos^2 B - \sin^2 A\cos^2 B - \sin^2 A + \cos^2 B\sin^2 A$

$= \cos^2 B - \sin^2 A$

$\boxed{\cos(A+B)\cos(A-B) = \cos^2 B - \sin^2 A}$

A = B रखने पर

$$\boxed{\cos 2A = \cos^2 A - \sin^2 A}$$

$$\boxed{\cos 2A = \cos^2 A - (1-\cos^2 A) = 2\cos^2 A - 1}$$

$$\boxed{\cos 2A = (1-\sin^2 A) - \sin^2 A = 1 - 2\sin^2 A}$$

$$\boxed{\cos 2A = \frac{\cos^2 A - \sin^2 A}{\sin^2 A + \cos^2 A} = \frac{1-\tan^2 A}{1+\tan^2 A}}$$

समी. ① ÷ समी. ③

$$\frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$$

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}} \quad — ⑤$$

समी. ② ÷ समी. ④

$$\frac{\sin(A-B)}{\cos(A-B)} = \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B + \sin A\sin B}$$

$$\boxed{\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}} \quad — ⑥$$

समी ⑤ से

$$\frac{1}{\tan(A+B)} = \frac{1 - \tan A\tan B}{\tan A + \tan B}$$

$$\boxed{\cot(A+B) = \frac{\cot A\cot B - 1}{\cot A + \cot B}}$$

समी. ⑥ से

$$\boxed{\cot(A-B) = \frac{\cot A\cot B + 1}{\cot B - \cot A}}$$

$\sqrt{1-\sin x}$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A}$$

$\sin(\overbrace{A+B}^{\alpha} + \overbrace{C}^{\beta}) = \sin(A+B)\cos(C) + \cos(A+B)\sin C$

$\cos(A+B+C) = \cos(A+B)\cos(C) - \sin(A+B)\sin C$

$\sin(60-\theta)\sin\theta\sin(60+\theta) = \frac{1}{4}\sin 3\theta$

$\cos(60-\theta)\cos\theta\cos(60+\theta) = \frac{1}{4}\cos 3\theta$

$\tan(60-\theta)\tan\theta\tan(60+\theta) = \tan 3\theta$

* $\cos A\cos 2A\cos 2^2 A \cdots\cdots \cos 2^{n-1} A = \dfrac{\sin 2^n A}{2^n \sin A}$

Exam $\cos \pi/7 \cos 2\cdot\frac{\pi}{7} \cos 2^2\frac{\pi}{7} = \dfrac{\sin 2^3\frac{\pi}{7}}{8\sin \pi/7} = \dfrac{\sin\frac{8\pi}{7}}{8\sin \pi/7}$

$= -\frac{1}{8}$

* $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots + \sin(\alpha+(n-1)\beta)$

$$= \frac{\sin\left[\alpha + \frac{(n-1)}{2}\beta\right]\sin\frac{n\beta}{2}}{\sin\beta/2}$$

* $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots + \cos(\alpha+(n-1)\beta)$

$$= \frac{\cos\left[\alpha + \frac{(n-1)}{2}\beta\right]\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}$$

$$\frac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$$

Q $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$ तब $\frac{\tan x}{\tan y}$

Ans
$\sin x\cos y + \cos x\sin y = a+b$
$\sin x\cos y - \cos x\sin y = a-b$
$2\sin x\cos y = 2a$
$\sin x\cos y = a$
$\cos x\cos y = b$
$\frac{\sin x\cos y}{\cos x\sin y} = \frac{a}{b}$ $\boxed{\frac{\tan x}{\tan y} = \frac{a}{b}}$

Q यदि x द्वितीय चतुर्थांश
$$\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$$

Ans
$$\frac{\sin\frac{x}{2}+\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}}{\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}+\cos\frac{x}{2}} = \frac{2\sin x/2}{2\cos x/2} = \tan\frac{x}{2}$$

Q $\sin A = n\sin B$
$\frac{n-1}{n+1}\tan\left(\frac{A+B}{2}\right) = ?$

Ans $\frac{\sin A-\sin B}{\sin A+\sin B} = \frac{n-1}{n+1}$

$$\frac{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)} = \frac{n-1}{n+1}$$

$\cot\left(\frac{A+B}{2}\right)\tan\left(\frac{A-B}{2}\right) = \frac{n-1}{n+1}$

$\left(\frac{n-1}{n+1}\right)\tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{A-B}{2}\right)$

त्रिकोणमितीय समीकरण और उसके हल

	$\sin\theta=0$	$\cos\theta=0$	$\tan\theta=0$	$\cot\theta=0$
मुख्य हल	$\theta=0$	$\theta=\pi/2$	$\theta=0$	$\theta=\pi/2$
व्यापक हल	$n\pi\ \forall n\in I$	$(2n+1)\ \forall n\in I$	$n\pi\ \forall n\in I$	$(2n+1)\forall n\in I$

		मुख्य हल	व्यापक हल
sin	$\sin\theta = a\ [a\in[-1,1]]$	α	$\theta = n\pi+(-1)^n\alpha\ \forall n\in I$
	$\cos\theta = a\ (a\in[-1,1])$	α	$\theta = 2n\pi\pm\alpha\ \forall n\in I$
	$\tan\theta = a\ (a\in(-\infty,\infty))$	α	$\theta = n\pi+\alpha\ \forall n\in I$
	$\cot\theta = a\ [a\in(-\infty,\infty)]$	α	$\theta = n\pi+\alpha\ \forall n\in I$
	$\sec\theta = a\ [a\in R-(-1,1)]$	α	$\theta = 2n\pi\pm\alpha\ \forall n\in I$
	$\operatorname{cosec}\theta = a\ [a\in R-(-1,1)]$	α	$\theta = n\pi+(-1)^n\alpha\ \forall n\in I$

1 $A+B+C = 180 \Rightarrow A+B = 180-C \Rightarrow C = 180-(A+B)$

(i) $\sin 2A + \sin 2B + \sin 2C =$
$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$
$= 2\sin\frac{2(A+B)}{2}\cos\frac{2(A-B)}{2} + \sin 2C$
$= 2\sin(A+B)\cos(A-B) + \sin 2C$
$= 2\sin(180-C)\cos(A-B) + \sin 2C$
$= 2\sin C\cos(A-B) + 2\sin C\cos C$
$= 2\sin C[\cos(A-B) + \cancel{\sin}\cos C]$
$= 2\sin C[\cos(A-B) + \cos(180-(A+B))]$
$= 2\sin C[\cos(A-B) - \cos(A+B)]$
$= 2\sin C[2\sin A\sin B]$
$= 4\sin A\sin B\sin C$

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(i) $\sin 2A + \sin 2B - \sin 2C =$

$2\sin(A+B)\cos(A-B) - \sin 2C$

$2\sin(180-C)\cos(A-B) - \sin 2C$

$2\sin C\cos(A-B) - 2\sin C\cos C$

$2\sin C[\cos(A-B) - \cos C]$

$2\sin C[\cos(A-B) - \cos(180-(A+B))]$

$2\sin C[\cos(A-B) + \cos(A+B)]$

$2\sin C[\cos A\cos B + \sin A\sin B + \cos A\cos B - \sin A\sin B]$

$2\sin C[2\cos A\cos B]$

$= 4\cos A\cos B\sin C$

(iii) $\sin(B+C-A) + \sin(C+A-B) + \sin(A+B-C)$

$= \sin(180-A-A) + \sin(180-B-B) + \sin(180-C-C)$

$= \sin 2A + \sin 2B + \sin 2C$ — ① के अनुसार हल

(iv) $\cos 2A + \cos 2B + \cos 2C =$

$2\cos\frac{2(A+B)}{2}\cos\frac{2(A-B)}{2} + \cos 2C$

$= 2\cos(A+B)\cos(A-B) + \cos 2C$

$= 2\cos(180-C)\cos(A-B) + \cos 2C$

$= -2\cos C\cos(A-B) + 2\cos^2 C - 1$

$= -2\cos C[\cos(A-B) - \cos C] - 1$

$= -2\cos C[\cos(A-B) - \cos(180-(A+B))] - 1$

$= -2\cos C[\cos(A-B) + \cos(A+B)] - 1$

$= -2\cos C[2\cos A\cos B] - 1$

$= -1 - 4\cos A\cos B\cos C$

(v) $\cos 2A + \cos 2B - \cos 2C =$

$2\cos(A+B)\cos(A-B) - 2\cos^2 C + 1 = -2\cos C\cos(A-B) - 2\cos^2 C + 1$

$-2\cos C[\cos(A-B) - \cos(180-(A+B))] + 1$

$-2\cos C[\cos(A-B) + \cos(A+B)] + 1$

$= 1 - 4\cos A\cos B\cos C$

② $A+B+C = 180°$

(i) $\sin(A) + \sin(B) + \sin(C) = 4\cos A/_2 \cos B/_2 \cos C/_2$

$2\sin\frac{(A+B)}{2}\cos\frac{(A-B)}{2} + \sin(C)$

$2\sin\left(\frac{180}{2} - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) + \sin C$

$2\cos\frac{C}{2}\cos\frac{(A-B)}{2} + 2\sin\frac{C}{2}\cos\frac{C}{2}$

$2\cos\frac{C}{2}\left[\cos\left(\frac{A}{2} - \frac{B}{2}\right) + \sin\frac{C}{2}\right]$

$2\cos\frac{C}{2}\left[\cos\left(\frac{A}{2} - \frac{B}{2}\right) + \sin\left(90 - \left(\frac{A+B}{2}\right)\right)\right]$

$= 2\cos\frac{C}{2}\left[\cos\left(\frac{A}{2} - \frac{B}{2}\right) + \cos\left(\frac{A}{2} + \frac{B}{2}\right)\right]$

$= 2\cos\frac{C}{2}\left[2\cos\frac{A}{2}\cos\frac{B}{2}\right] = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(ii) $\sin A + \sin B - \sin C = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) - \sin C$

$2\cos\frac{C}{2}\cos\left(\frac{A-B}{2}\right) - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$2\cos\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \sin\left(90 - \left(\frac{A+B}{2}\right)\right)\right]$

$2\cos\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right]$

$= 2\cos\frac{C}{2}\left[2\sin\frac{A}{2}\sin B/_2\right] = 4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

(iii) $\cos A + \cos B + \cos C = 1 + 4\sin A/_2 \sin B/_2 \sin C/_2$

$\cancel{2\cos\frac{C}{2}\cos\left(\frac{A-B}{2}\right) + 2\sin\cos^2\frac{C}{2}} -$

$2\cos\frac{A+B}{2}\cos\left(\frac{A-B}{2}\right) + \cos C$

$$2\cos\left(90-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right)+1-2\sin^2\frac{C}{2}$$

$$2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)+1-2\sin^2\frac{C}{2}$$

$$2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)-\sin\frac{C}{2}\right]+1$$

$$2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)-\sin\left(90-\left(\frac{A+B}{2}\right)\right)\right]+1$$

$$2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{A+B}{2}\right)\right]+1$$

$$2\sin\frac{C}{2}\left[2\sin\frac{A}{2}\sin\frac{B}{2}\right]+1$$

$$=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

(iv) $\cos A+\cos B-\cos C=-1+4\cos\frac{A}{2}\cos B/_2\sin C/_2$

$$2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)-\cos C=2\cos\left(90-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right)-\cos C$$

$$2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)-\cancel{2\sin^2\frac{C}{2}}+\left[1-2\sin^2\frac{C}{2}\right]$$

$$=2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)+\sin\frac{C}{2}\right]-1$$

$$=2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)+\cos\left(\frac{A+B}{2}\right)\right]-1$$

$$=2\sin\frac{C}{2}\left[2\cos\frac{A}{2}\cos\frac{B}{2}\right]-1$$

$$=4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}-1$$

$$=-1+4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

(v) $\frac{\cos A}{\sin B\sin C}+\frac{\cos B}{\sin C\sin A}+\frac{\cos C}{\sin \sin B}=2$

$$=\frac{\cos A\sin A+\sin B\cos B+\sin C\cos C}{\sin A\sin B\sin C}$$

$$=\frac{\frac{1}{2}\sin 2A+\frac{1}{2}\sin 2B+1/_2\sin 2C}{\sin A\sin B\sin C}$$

$$=\frac{\frac{1}{2}\left[\sin 2A+\sin 2B+\sin 2C\right]}{\sin A\sin B\sin C}$$

$$=\frac{\frac{1}{2}\left[4\sin A\sin B\sin C\right]}{\sin A\sin B\sin C}=2$$

(3) $A+B+C=\pi$ दिया

(i) $\sin^2 A+\sin^2 B-\sin^2 C$

$$=\frac{1-\cos 2A}{2}+\frac{1-\cos 2B}{2}-\left(\frac{1-\cos 2C}{2}\right)$$

$$=\frac{1}{2}-\frac{\cos 2A}{2}+\frac{1}{2}-\frac{\cos 2B}{2}-\frac{1}{2}+\frac{\cos 2C}{2}$$

$$=\frac{1}{2}-\frac{1}{2}\left[\cos 2A+\cos 2B-\cos 2C\right]$$

$$=\frac{1}{2}-\frac{1}{2}\left[1-4\sin A\sin B\cos C\right]$$

$$=\frac{1}{2}-\frac{1}{2}+2\sin A\sin B\cos C=2\sin A\sin B\cos C$$

(ii) $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=\frac{1}{2}+\frac{\cos 2A}{2}+\frac{1}{2}+\frac{\cos 2B}{2}+\frac{1}{2}+\frac{\cos C}{2}$

$$=\frac{3}{2}+\frac{1}{2}\left[\cos 2A+\cos 2B+\cos 2C\right]$$

$$=3/_2+1/_2\left[-1-4\cos A\cos B\cos C\right]$$

$$=3/_2-1/_2-2\cos A\cos B\cos C=1-2\cos A\cos B\cos C$$

(iii) $\sin^2 A + \sin^2 B + \sin^2 C = 1 - 2\sin A \sin B \cos C$

$\frac{1}{2} - \frac{\cos 2A}{2} + \frac{1}{2} - \frac{\cos 2B}{2} + \frac{1}{2} - \frac{\cos 2C}{2}$

$= \frac{3}{2} - \frac{1}{2}\left[\cos 2A + \cos 2B + \cos 2C\right]$

$= \frac{3}{2} - \frac{1}{2}\left[-1 - 4\cos A \cos B \cos C\right]$

$= 1 + 2\cos A \cos B \cos C$

$=$

(4) यदि $A+B+C = \pi$

(i) $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2}$

$= \sin^2 \frac{A}{2} + \left(1 - \sin^2 \frac{B}{2}\right) + \sin^2 \frac{C}{2}$

$= \sin^2 \frac{A}{2} + \cos^2 \frac{B}{2} + \sin^2 \frac{C}{2} + 1$

$= \cancel{\cos}\ \frac{1-\cos A}{2} + \frac{1-\cos B}{2} + \frac{1-\cos C}{2}$

$= \frac{3}{2} - \frac{1}{2}\left[\cos A + \cos B + \cos C\right]$

$= \frac{3}{2} - \frac{1}{2}\left[1 + 4\sin \frac{A}{2} \sin B/2 \sin C/2\right]$

$= 1 - 2\sin \frac{A}{2} \sin B/2 \sin C/2$

(ii) $\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} + \cos^2 \frac{C}{2}$

$= \frac{3}{2} + 1/2\left[\cos A + \cos B + \cos C\right]$

$= \frac{3}{2} + \frac{1}{2}\left[1 + 4\sin \frac{A}{2} \sin B/2 \sin C/2\right]$

$= 2 + 2\sin \frac{A}{2} \sin B/2 \sin C/2$

* यदि $\sin^2\theta = \sin^2\alpha$
$\cos^2\theta = \cos^2\alpha$
$\tan^2\theta = \tan^2\alpha$
$\cot^2\theta = \cot^2\alpha$
$\sec^2\theta = \sec^2\alpha$
$\operatorname{cosec}^2\theta = \operatorname{cosec}^2\alpha$

= व्यापक हल $\theta = n\pi \pm \alpha$

$\sin^2\theta = a \Rightarrow \sin^2\theta = \sin^2\alpha$

$$\left(\frac{1-\cos 2\theta}{2}\right) = \left(\frac{1-\cos 2\alpha}{2}\right)$$

$$\cos 2\theta - \cos 2\alpha = 0$$

$$-2\sin(\theta+\alpha)\sin(\theta-\alpha) = 0$$

$\theta+\alpha = 0 \qquad \theta-\alpha = 0$

$\theta+\alpha = n\pi \qquad \theta-\alpha = n\pi$

$\theta = n\pi-\alpha \qquad \theta = n\pi+\alpha$

Q. $\frac{1-\cos 2\theta}{1+\cos 2\theta} = 3$

$\frac{2\sin^2\theta}{2\cos^2\theta} = 3 \Rightarrow \tan^2\theta = 3 \Rightarrow \tan^2\theta = (\sqrt{3})^2$

$\tan^2\theta = \tan^2(\pi/3)$ then $\theta = n\pi \pm \pi/3$

Q. $\sin^2\theta = 1$ तो कौनसा सत्य नहीं है -

(i) $\theta = n\pi + \pi/2$ –
(ii) $\theta = n\pi - \pi/2$ –
(iii) $\theta = 2n\pi + \pi/2$ –
(iv) $\theta = 2n\pi \pm \pi/2$ –

$\sin^2\theta = \sin^2\pi/2$
$\theta = n\pi \pm \pi/2$

Q. $\cos\theta + 2\cos 2\theta + \cos 3\theta = 0$

(i) $\theta = 2n\pi \pm 2\pi/3$
(ii) $\theta = 2n\pi \pm \pi/4$
(iii) $n\pi + (-1)^n \pi/3$

$\cos\theta + \cos 2\theta + \cos 3\theta = 0$

$\cos\theta + \cos 3\theta + \cos 2\theta = 0$

$2\cos 2\theta \cos\theta + \cos 2\theta = 0$

$\cos 2\theta(2\cos\theta + 1) = 0$

$\cos 2\theta = 0$
$2\theta = (2n+1)\frac{\pi}{2}$
$\theta = (2n+1)\pi/4$

$2\cos\theta + 1 = 0$
$\cos\theta = -1/2$
$\cos\theta = \cos\frac{2\pi}{3}$

$\theta = 2n\pi \pm \frac{2\pi}{3}$

Q. $\sec^2\theta = 4/3$ θ का व्यापक मान

Ans $\sec^2\theta = \left(\frac{2}{\sqrt{3}}\right)^2 \Rightarrow \theta = n\pi \pm \pi/6$

Q. $3(\sec^2\theta + \tan^2\theta) = 5$

$\sec^2\theta + \tan^2\theta = \frac{5}{3}$

$1 + \tan^2\theta + \tan^2\theta = \frac{5}{3} \Rightarrow 2\tan^2\theta = \frac{5}{3} - 1 = 2/3$

$\tan^2\theta = 1/3 \Rightarrow \tan^2\theta = \tan^2\pi/6$

$\theta = n\pi \pm \pi/6$

Q. $\cot\theta - \tan\theta = 2 \Rightarrow \frac{1}{\tan\theta} - \tan\theta = 2$

$1 - \tan^2\theta = 2\tan\theta$

Ans $\cos 2\theta = \sin 2\theta \Rightarrow \tan 2\theta = 1$

$\tan 2\theta = \tan\pi/4 \Rightarrow 2\theta = n\pi + \pi/4$

$$\boxed{\theta = \frac{n\pi}{2} + \frac{\pi}{8}}$$

$-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$

* युगपत समीकरण का व्यापक हल ज्ञात करना :-
सर्वप्रथम दि गयी युगपत समीकरणों मे से प्रत्येक समीकरण को सन्तुष्ट करने वाले 0–360° के मध्य सन्तुष्ट करने वाले कोण ज्ञात करते है उन कोणों मे से उभयनिष्ट कोण को $2n\pi$ में जोड़ने से सभी का व्यापक हल प्राप्त होता है।

Q युगपत समी. $\sin\theta = -\sqrt{3}/2$, $\cos\theta = -1/2$ का व्यापक हल

Ans $\sin\theta = -\frac{\sqrt{3}}{2}$ $\qquad$ $\cos\theta = -1/2$

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ $\qquad$ $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ $\qquad$ $\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

व्यापक हल $= 2n\pi + \frac{4\pi}{3}$

Q $3\sin x + 4\cos x = 2K+1$ का एक हल होगा?

Ans $-\sqrt{9+16} \le 3\sin x + 4\cos x \le \sqrt{9+16}$

$-\sqrt{25} \le 3\sin x + 4\cos x \le \sqrt{25}$

$-5 \le 3\sin x + 4\cos x \le 5$

$-5 \le 2K+1 \le 5 \Rightarrow -6 \le 2K \le 4$

$-3 \le K \le 2$ $\qquad$ $-3 \; -2 \; -1 \; 0 \; 1 \; 2$ $\quad$ 6

अतः 6 मान होंगे

Q K के पूर्णांक मानों की संख्या कितनी है जिसके लिए

$7\cos x + 5\sin x = 2K+1$ का एक हल होगा।

Ans $-\sqrt{49+25} \le 7\cos x + 5\sin x \le \sqrt{49+25}$

$-\sqrt{74} \le 2K+1 \le \sqrt{74}$

$-8.6 \le 2K+1 \le 8.6$

$-9.6 \le 2K \le 7.6$

$-4.8 \le K < 3.8$ $\qquad$ $-4 \; -3 \; -2 \; -1 \; 0 \; 1 \; 2 \; 3$

8 मान

$-\frac{\pi}{3} - \frac{\pi}{6}$

* $a\sin\theta + b\cos\theta = c$ रूप की त्रिकोणमितिय समीकरण का व्यापक हल ज्ञात करना

$a\sin\theta + b\cos\theta = c$

$\sqrt{a^2+b^2}$ का भाग देने पर

$\frac{a}{\sqrt{a^2+b^2}}\sin\theta + \frac{b}{\sqrt{a^2+b^2}}\cos\theta = \frac{c}{\sqrt{a^2+b^2}}$

$\cos\theta\cos\alpha + \sin\theta\sin\alpha = \cos\beta$

$\cos(\theta-\alpha) = \cos\beta$

$\theta - \alpha = 2n\pi \pm \beta$

$\boxed{\theta = 2n\pi \pm \beta + \alpha}$

Q $\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \sqrt{3}/2$

Ans $\cos\frac{\pi}{6}\sin x + \sin\frac{\pi}{6}\cos x = \sin\frac{\pi}{3}$

$\sin(x + \pi/6) = \sin\pi/3$

$x + \frac{\pi}{6} = n\pi + (-1)^n\frac{\pi}{3}$

$x = n\pi + (-1)^n\frac{\pi}{3} - \frac{\pi}{6}$

Or $\cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x = \cos\frac{\pi}{6}$

$\cos\left(x - \frac{\pi}{3}\right) = \cos\frac{\pi}{6} \Rightarrow x - \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{6}$

$x = 2n\pi \pm \frac{\pi}{6} + \frac{\pi}{3}$

(+) चिह्न लेने पर $\quad x = 2n\pi + \pi/2$

(−) चिह्न लेने पर $\quad x = 2n\pi + \pi/6$