

Exemplar Problem

Trigonometric Functions

11. If $\tan(A + B) = p$, $\tan(A - B) = q$, then show that $\tan 2A = (p + q) / (1 - pq)$.

[Hint: Use $2A = (A + B) + (A - B)$]

Solution:

We know that,

$$\tan 2A = \tan(A + B + A - B)$$

And also,

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B)\tan(A-B)}$$

$$\therefore \tan 2A = \frac{p + q}{1 - pq}$$

Substituting the values given in question,

$$\Rightarrow \tan 2A = \frac{p + q}{1 - pq}$$

$$\text{Hence, } \tan 2A = \frac{p + q}{1 - pq}$$