

## Exemplar Problem

### Trigonometric Functions

8. Find the value of  $\tan 22^\circ 30'$ .

[Hint: Let  $\theta = 45^\circ$ , use  $\tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} = \frac{\sin \theta}{1 + \cos \theta}$ ]

**Solution:**

Let,  $\theta = 45^\circ$

As we need to find:  $\tan 22^\circ 30' = \tan (\theta/2)$

We know that,

$$\sin \theta = \cos \theta = 1/\sqrt{2} \text{ (for } \theta = 45^\circ \text{)}$$

Since,

$$\tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}}$$

Multiplying  $2 \cos \frac{\theta}{2}$  in numerator and denominator, we get,

$$\Rightarrow \tan \frac{\theta}{2} = \frac{2 \cos \frac{\theta}{2} \sin \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}}$$

By applying formula of T-ratios of multiple angles-

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = 2 \cos^2 x - 1 \text{ or } 1 + \cos 2x = 2 \cos^2 x$$

$$\therefore \tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}$$

$$\Rightarrow \tan 22^\circ 30' = \frac{\sin 45^\circ}{1 + \cos 45^\circ}$$

$$= \frac{\frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}}$$
$$= \frac{1}{\sqrt{2} + 1}$$

By rationalizing the term, we get,

$$\Rightarrow \tan 22^\circ 30' = \frac{1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{\sqrt{2} - 1}{(\sqrt{2})^2 - 1}$$

Therefore,  $\tan 22^\circ 30' = \sqrt{2} - 1$