5. If tanx = b/a then find the value of

$$\sqrt{\frac{a+b}{a-b}}+\sqrt{\frac{a-b}{a+b}}.$$

Solution:

According to the question,

tan x = b/a

Let,

$$y=\sqrt{\frac{a+b}{a-b}}+\sqrt{\frac{a-b}{a+b}}$$

$$\therefore y=\sqrt{\frac{a\left(1+\frac{b}{a}\right)}{a\left(1-\frac{b}{a}\right)}}+\sqrt{\frac{a\left(1-\frac{b}{a}\right)}{a\left(1+\frac{b}{a}\right)}}$$

$$=\sqrt{\frac{(1+\tan x)}{(1-\tan x)}}+\sqrt{\frac{(1-\tan x)}{(1+\tan x)}}$$

$$=\frac{\sqrt{1+\tan x}}{\sqrt{1-\tan x}}+\frac{\sqrt{1-\tan x}}{\sqrt{1+\tan x}}$$

$$=\frac{\left(\sqrt{1+\tan x}\right)^2+\left(\sqrt{1-\tan x}\right)^2}{\left(\sqrt{1-\tan x}\right)\left(\sqrt{1+\tan x}\right)}$$

$$=\frac{1+\tan x+1-\tan x}{\sqrt{1-\tan^2 x}}=\frac{2}{\sqrt{1-\tan^2 x}}$$

$$\therefore y=\sqrt{\frac{a+b}{a-b}}+\sqrt{\frac{a-b}{a+b}}=\frac{2}{\sqrt{1-\tan^2 x}}$$

$$=\frac{2}{\sqrt{1-\frac{\sin^2\theta}{\cos^2\theta}}}$$

$$=\frac{2}{\frac{\sqrt{\cos^2\theta-\sin^2\theta}}{\cos\theta}}$$

$$\because \cos^2\theta-\sin^2\theta=\cos 2\theta$$

$$=\frac{2\cos\theta}{\sqrt{\cos 2\theta}}$$