

Exemplar Problem

Trigonometric Functions

5. If $\tan x = b/a$ then find the value of

$$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$$

Solution:

According to the question,

$$\tan x = b/a$$

Let,

$$\begin{aligned} y &= \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} \\ \therefore y &= \sqrt{\frac{a(1+\frac{b}{a})}{a(1-\frac{b}{a})}} + \sqrt{\frac{a(1-\frac{b}{a})}{a(1+\frac{b}{a})}} \\ &= \sqrt{\frac{(1+\tan x)}{(1-\tan x)}} + \sqrt{\frac{(1-\tan x)}{(1+\tan x)}} \\ &= \frac{\sqrt{1+\tan x}}{\sqrt{1-\tan x}} + \frac{\sqrt{1-\tan x}}{\sqrt{1+\tan x}} \\ &= \frac{(\sqrt{1+\tan x})^2 + (\sqrt{1-\tan x})^2}{(\sqrt{1-\tan x})(\sqrt{1+\tan x})} \\ &= \frac{1+\tan x + 1-\tan x}{\sqrt{1-\tan^2 x}} = \frac{2}{\sqrt{1-\tan^2 x}} \\ \therefore y &= \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{2}{\sqrt{1-\tan^2 x}} \\ &= \frac{2}{\sqrt{1-\frac{\sin^2 \theta}{\cos^2 \theta}}} \\ &= \frac{2}{\sqrt{\cos^2 \theta - \sin^2 \theta}} \\ \therefore \cos^2 \theta - \sin^2 \theta &= \cos 2\theta \\ &= \frac{2\cos \theta}{\sqrt{\cos 2\theta}} \end{aligned}$$