Exemplar Problem

Trigonometric Functions

1. Prove that

$$\frac{\tan A+\sec A-1}{\tan A-\sec A+1}=\frac{1+\sin A}{\cos A}$$

Solution:

According to the question,

$$\text{LHS} = \frac{\tan A+\sec A-1}{\tan A-\sec A+1}$$
$$= \frac{\frac{\sin A}{\cos A}+\frac{1}{\cos A}-1}{\frac{\sin A}{\cos A}-\frac{1}{\cos A}+1}$$
$$= \frac{\sin A+1-\cos A}{\sin A-1+\cos A}$$
$$= \frac{\sin A+(1-\cos A)}{\sin A-(1-\cos A)}$$

Using the identity,

$\sin^2 A + \cos^2 A = 1$, we get,

sin A + (1 − cos A).

$$\therefore \text{LHS} = \frac{\sin A+(1-\cos A)}{\sin A-(1-\cos A)} \times \frac{\sin A+(1-\cos A)}{\sin A+(1-\cos A)}$$
$$= \frac{\{\sin A+(1-\cos A)\}^2}{\sin^2 A-(1-\cos A)^2}$$
$$= \frac{\sin^2 A+(1-\cos A)^2+2\sin A\,(1-\cos A)}{\sin^2 A-(1-\cos A)^2}$$
$$= \frac{(\sin^2 A+\cos^2 A)+1-2\cos A+2\sin A\,(1-\cos A)}{\sin^2 A-\{1+\cos^2 A-2\cos A\}}$$
$$= \frac{(1)+1-2\cos A+2\sin A\,(1-\cos A)}{(\sin^2 A-1)-\cos^2 A+2\cos A}$$
$$= \frac{2(1-\cos A)+2\sin A\,(1-\cos A)}{(-\cos^2 A)-\cos^2 A+2\cos A}$$
$$= \frac{2(1+\sin A)(1-\cos A)}{-2\cos^2 A+2\cos A}$$
$$= \frac{2(1+\sin A)(1-\cos A)}{2\cos A\,(1-\cos A)}$$
$$= \frac{(1+\sin A)}{\cos A} = \text{RHS}$$

Hence, L.H.S = R.H.S