12. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then prove that $\cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$.
[Hint: $\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2 = 0$]

Solution:

According to the question,

$\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$ …(i)

Since, LHS = $\cos 2\alpha + \cos 2\beta$

We know that,

$\cos 2x = \cos^2 x - \sin^2 x$

Therefore,

LHS = $\cos^2\alpha - \sin^2\alpha + (\cos^2\beta - \sin^2\beta)$

$\Rightarrow$ LHS = $\cos^2\alpha + \cos^2\beta - (\sin^2\alpha + \sin^2\beta)$

Also, since,

$a^2 + b^2 = (a+b)^2 - 2ab$

$\Rightarrow$ LHS = $(\cos\alpha + \cos\beta)^2 - 2\cos\alpha\cos\beta - (\sin\alpha + \sin\beta)^2 + 2\sin\alpha\sin\beta$

From equation (i),

$\Rightarrow$ LHS = $0 - 2\cos\alpha\cos\beta - 0 + 2\sin\alpha\sin\beta$

$\Rightarrow$ LHS = $-2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$

$\because \cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Therefore, LHS = $-2\cos(\alpha + \beta)$ = RHS

Hence, $\cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$