

Exemplar Problem

Trigonometric Functions

9. Prove that $\sin 4A = 4\sin A \cos^3 A - 4\cos A \sin^3 A$.

Solution:

$$\sin 4A = \sin (2A + 2A)$$

We know that,

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\text{Therefore, } \sin 4A = \sin 2A \cos 2A + \cos 2A \sin 2A$$

$$\Rightarrow \sin 4A = 2 \sin 2A \cos 2A$$

From T-ratios of multiple angle,

We get,

$$\sin 2A = 2 \sin A \cos A \text{ and } \cos 2A = \cos^2 A - \sin^2 A$$

$$\Rightarrow \sin 4A = 2(2 \sin A \cos A)(\cos^2 A - \sin^2 A)$$

$$\Rightarrow \sin 4A = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$$

$$\text{Hence, } \sin 4A = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$$