7. If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, then show that $a^2 + b^2 = m^2 + n^2$.

Solution:

According to the question,

$a \cos \theta + b \sin \theta = m$ ...(i)

$a \sin \theta - b \cos \theta = n$ ...(ii)

Squaring and adding equation 1 and 2, we get,

$(a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2 = m^2 + n^2$

$\Rightarrow a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = m^2 + n^2$

$\Rightarrow a^2 \cos^2 \theta + b^2 \sin^2 \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta = m^2 + n^2$

$\Rightarrow a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = m^2 + n^2$

Using, $\sin^2 \theta + \cos^2 \theta = 1$,

We get,

$\Rightarrow a^2 + b^2 = m^2 + n^2$