6. Prove that cos θ cos θ/2 – cos 3θ cos 9θ/2 = sin7θ sin4θ
[Hint: Express L.H.S. = ½ [2cos θcos θ/2 – 2cos 3θ cos 9θ / 2]

Solution:

Using transformation formula, we get,

2 cos A cos B = cos(A + B) + cos (A – B)

-2 sin A sin B = cos(A + B) – cos (A – B)

Multiplying and dividing the expression by 2.

$$\therefore \text{LHS} = \frac{1}{2}\left(2\cos\theta\cos\frac{\theta}{2} - 2\cos 3\theta\cos\frac{9\theta}{2}\right)$$

Applying transformation formula, we get,

$$\text{LHS} = \frac{1}{2}\left(\cos\left(\theta + \frac{\theta}{2}\right) + \cos\left(\theta - \frac{\theta}{2}\right) - \left\{\cos\left(3\theta + \frac{9\theta}{2}\right) + \cos\left(3\theta - \frac{9\theta}{2}\right)\right\}\right)$$

$$\Rightarrow \text{LHS} = \frac{1}{2}\left(\cos\frac{3\theta}{2} + \cos\frac{\theta}{2} - \cos\left(\frac{15\theta}{2}\right) - \cos\left(-\frac{3\theta}{2}\right)\right)$$

$$\Rightarrow \text{LHS} = \frac{1}{2}\left(\cos\frac{3\theta}{2} + \cos\frac{\theta}{2} - \cos\frac{15\theta}{2} - \cos\frac{3\theta}{2}\right) \{\because \cos(-x) = \cos x\}$$

$$\Rightarrow \text{LHS} = \frac{1}{2}\left(\cos\frac{\theta}{2} - \cos\frac{15\theta}{2}\right)$$

$$\Rightarrow \text{LHS} = \frac{1}{2}\left(2\sin\left(\frac{\frac{\theta}{2}+\frac{15\theta}{2}}{2}\right)\sin\left(\frac{\frac{15\theta}{2}-\frac{\theta}{2}}{2}\right)\right)$$

$$\Rightarrow \text{LHS} = \frac{1}{2}\left(2\sin\left(\frac{8\theta}{2}\right)\sin\left(\frac{7\theta}{2}\right)\right)$$

$$\therefore \text{LHS} = \sin 4\theta \sin\left(\frac{7\theta}{2}\right) = \text{RHS}$$

Hence,

$$\cos\theta\cos\frac{\theta}{2} - \cos 3\theta\cos\frac{9\theta}{2} = \sin 4\theta \sin\left(\frac{7\theta}{2}\right)$$