2. If [2sinα / (1+cosα+sinα)] = y, then prove that [(1– cosα+sinα) / (1+sinα)] is also equal to y.

$$\left[\text{Hint : Express } \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} \cdot \frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}\right]$$

Solution:

According to the question,

y =2sinα /(1+cosα+sinα)

Multiplying numerator and denominator by (1 – cos α + sin α),

We get,

$$\Rightarrow y = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} \times \frac{1-\cos\alpha+\sin\alpha}{1-\cos\alpha+\sin\alpha}$$

$$= \frac{2\sin\alpha}{(1+\sin\alpha)+\cos\alpha} \times \frac{(1+\sin\alpha)-\cos\alpha}{(1+\sin\alpha)-\cos\alpha}$$

Using (a + b) (a-b) = $a^2 - b^2$, we get:

$$= \frac{2\sin\alpha\{(1+\sin\alpha)-\cos\alpha\}}{(1+\sin\alpha)^2-\cos^2\alpha}$$

$$= \frac{2\sin\alpha(1+\sin\alpha)-2\sin\alpha\cos\alpha}{1+\sin^2\alpha+2\sin\alpha-\cos^2\alpha}$$

Since, $1-\cos^2\alpha = \sin^2\alpha$

$$\therefore y = \frac{2\sin\alpha(1+\sin\alpha-\cos\alpha)}{\sin^2\alpha+2\sin\alpha+\sin^2\alpha}$$

$$= \frac{2\sin\alpha(1+\sin\alpha-\cos\alpha)}{2\sin\alpha(1+\sin\alpha)}$$

$$\Rightarrow y = \frac{(1+\sin\alpha-\cos\alpha)}{(1+\sin\alpha)}$$

$$\Rightarrow y = \frac{(1-\cos\alpha+\sin\alpha)}{(1+\sin\alpha)}$$

Hence Proved