Exemplar Problem

Trigonometry Functions

Example 16: If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, then a, b and c satisfy the relation.

a) $a^2 + b^2 + 2ac = 0$

b) $a^2 - b^2 + 2ac = 0$

c) $a^2 + c^2 + 2ab = 0$

d)

$$a^2 - b^2 - 2ac = 0$$

Ans: The correct answer is option (b) $a^2 - b^2 + 2ac = 0$

Given that, $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$.

We know that if the roots of the quadratic equation $ax^2 + bx + c = 0$ are α and β. Then we have, $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$. Therefore, we get

$\Rightarrow \sin\theta + \cos\theta = \frac{b}{a}...(i)$ and $\sin\theta\cos\theta = \frac{c}{a}.....(ii)$

On squaring both the sides in equation (i), we get

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{b^2}{a^2}$

We have, $\sin\theta\cos\theta = \frac{c}{a}$ and we know that $\sin^2\theta + \cos^2\theta = 1$. Therefore, we get

$\Rightarrow 1 + \frac{2c}{a} = \frac{b^2}{a^2}$

$\Rightarrow \frac{a + 2c}{a} = \frac{b^2}{a^2}$

$\Rightarrow \frac{a + 2c}{1} = \frac{b^2}{a}$

On cross multiplication, we get

$\Rightarrow a^2 + 2ac = b^2$

$\Rightarrow a^2 - b^2 + 2ac = 0$