

5. Find the coefficient of x^4 in the expansion of $(x/2 - 3/x^2)^{10}$

Solution:

In the expansion of $(x/2 - 3/x^2)^{10}$, the general term is:

$$\begin{aligned} T_{r+1} &= {}^{10}C_r \left(\frac{x}{2}\right)^{10-r} \left(-\frac{3}{x^2}\right)^r \\ &= {}^{10}C_r (-1)^r \frac{3^r}{2^{10-r}} x^{10-3r} \end{aligned}$$

Here, the exponent of x is

$$10 - 3r = 4 \Rightarrow r = 2.$$

$$\begin{aligned} \therefore T_{2+1} &= {}^{10}C_2 \left(\frac{x}{2}\right)^8 \left(-\frac{3}{x^2}\right)^2 \\ &= \frac{10 \times 9}{1 \times 2} \times \frac{1}{2^8} \times 3^2 \times x^4 \\ &= \frac{405}{256} x^4 \end{aligned}$$

Therefore, the required coefficient is $\boxed{405/256}$.