3) Find the coefficient of x^k in $1+(1+x)+(1+x)^2+\ldots+(1+x)^n$ $(0 \le k \le n)$.

Solution:

The expression being in G.P., we have

$$E = 1+(1+x)+(1+x)^2+\ldots\ldots+(1+x)^n \quad \cancel{(0 \le k \le n)}$$

$$= \frac{(1+x)^{n+1}-1}{(1+x)-1} = x^{-1}\left[(1+x)^{n+1}-1\right]$$

Therefore, the coefficient of x^k in E is equal to the coefficient of x^{k+1} in $\left[(1+x)^{n+1}-1\right]$, which is given by $\boxed{{}^{n+1}C_{k+1}}$