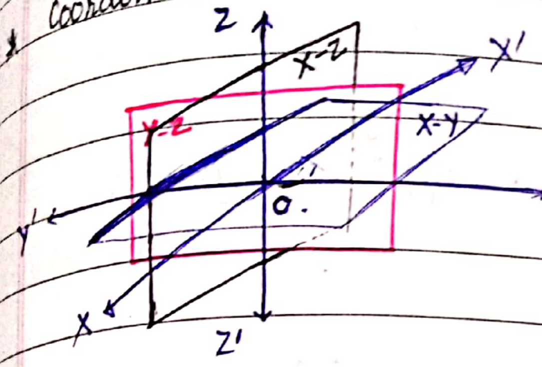
# THREE-DIMENSIONAL GEOMETRY

## * Coordinate Axes and Planes




in X-Z plane $y=0$

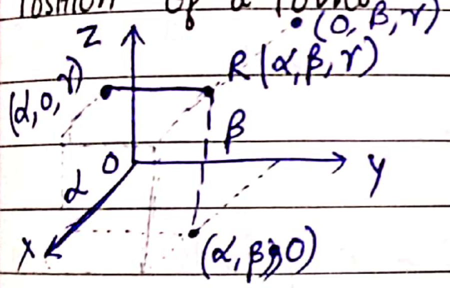
Y-Z plane $x=0$

X-Y plane $z=0$.

There are '<u>8 octants</u>'.

| Octant | OXYZ (I) | OX'YZ (II) | OX'Y'Z (III) | OXY'Z (IV) | OXYZ' (V) | OX'YZ' (VI) | OX'Y'Z' (VII) | OXY'Z' (VIII) |
|---|---|---|---|---|---|---|---|---|
| x | + | − | − | + | + | − | − | + |
| y | + | + | − | − | + | + | − | − |
| z | + | + | + | + | − | − | − | − |

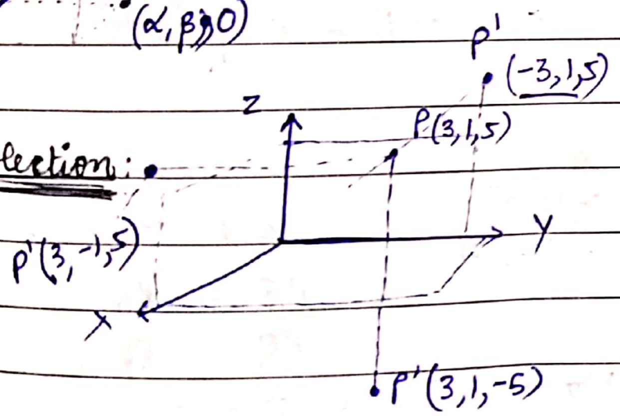
## * Position of a Point



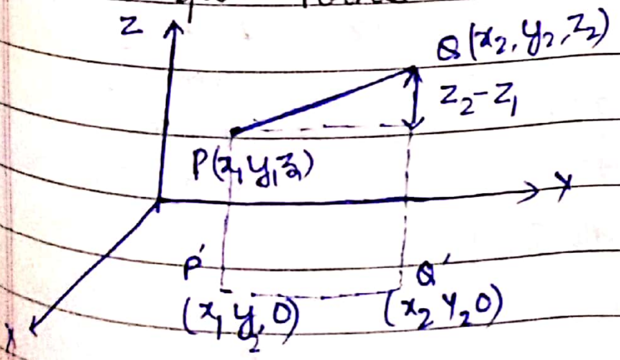

Ex. $(3, -6, 4)$ IV octant

$(-4, -3, 2)$ III

**Reflection:**




## * Distance b/w Points




$$P'Q' = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Now $$PQ = \sqrt{(P'Q')^2+(z_2-z_1)^2}$$

$$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

units.

Ex. $(3,1,0)$ and $(4,-1,2)$

$\sqrt{(3-4)^2+(2)^2+(2)^2} = \boxed{3 \text{ units}}$

Applications : Nature of Triangle
checking collinearity
Checking the type of figure

Ex. $A(0,7,10)$ $B(-1,6,6)$ $C(-4,9,6)$

$BC = \sqrt{3^2+3^2+0} = \sqrt{18}$

$AB = \sqrt{1^2+1^2+4^2} = \sqrt{18}$

$AC = \sqrt{16+4+16} = 6$

right angled Isosceles.

Ex. $(a,b,c), (c,a,b), (b,c,a)$ Equilateral $\Delta$.

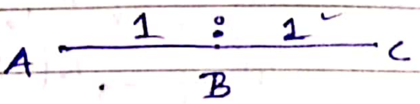
Ex. $A(3,-2,4)$ $B(1,0,-2)$ $C(-1,2,-8)$

$AB = \sqrt{4+4+36} = \sqrt{44} = 2\sqrt{11}$

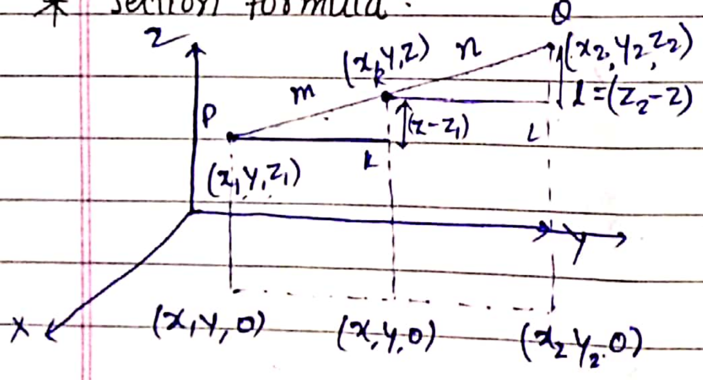
$BC = \sqrt{4+4+36} = \sqrt{44} = 2\sqrt{11}$

$AC = \sqrt{16+16+144} = \sqrt{176} = 4\sqrt{11}$

$AB + BC = AC$ $\Rightarrow$ collinear points.




* Section formula :




$\Delta PRK \sim \Delta QRL$ (similar)

$\frac{PR}{RQ} = \frac{RK}{QL}$

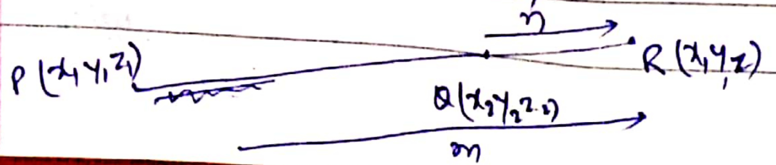
$\Rightarrow \frac{m}{n} = \frac{z - z_1}{z_2 - z}$

$\Rightarrow \boxed{z = \frac{mz_2 + nz_1}{m+n}}$

similarly, $y = \frac{my_2 + ny_1}{m+n}$ , $x = \frac{mx_2 + nx_1}{m+n}$

[for Internal division]

* If division is external




$\frac{m}{n} = \frac{PR}{RQ}$

$x = \frac{mx_2 - nx_1}{m-n}$ , $y = \frac{my_2 - ny_1}{m-n}$ , $z = \frac{mz_2 - nz_1}{m-n}$

1) $P(4,6,2)$ $Q(4,4,-8)$ $R$ 1) $\frac{PR}{RQ}$ internally in 2:1.

$R = \left(\frac{2\times4+1\times4}{3}, \frac{2\times4+1\times6}{3}, \frac{2\times-8+1\times2}{3}\right) = \left(4, \frac{14}{3}, \frac{-14}{3}\right)$.

2) externally $R = \left(\frac{2\times4-1\times4}{1}, \frac{2\times4-1\times6}{1}, \frac{2\times-8-1\times2}{1}\right)$

$= (4, 2, -18)$

* Applications of section formula.

(1) collinearity. $P(2,0,-1)$ $Q(1,-1,-3)$ $R(\lambda,\mu,3)$

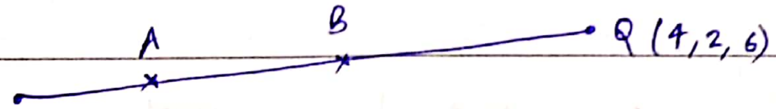
let R divides PQ in $m:1$.

$3 = \frac{-3m-1}{m+1} \Rightarrow 3m+3 = -3m-1 \Rightarrow m = -2/3$

$\lambda = \frac{-\frac{2}{3}+2}{-\frac{2}{3}+1} = 4$ $\mu = \frac{2/3}{-\frac{2}{3}+1} = 2$ $\boxed{R(4,2,3)}$

(2) Trisection

(4,8,2) P




A divides PQ in 1:2 ; B divides PQ in 2:1.

4) $P(4,4,-10)$, $Q(-2,2,4)$ find ratio in which x-y plane divides PQ.

x-y plane $z=0$ let ratio is $\lambda:1$

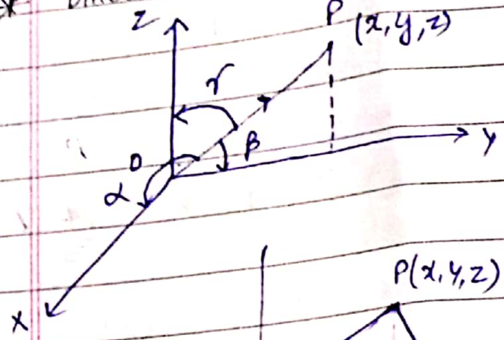
$0 = \frac{4\lambda-10}{\lambda+1} \Rightarrow \lambda = 5/2$

ratio = $\boxed{5:2}$

(3) Centroid - $G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

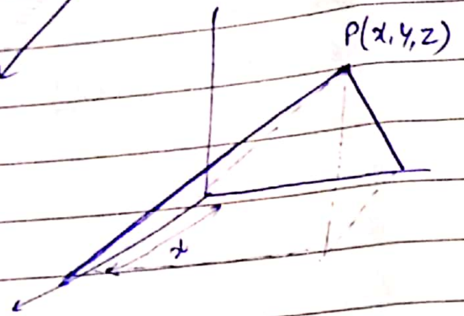
length of medians.

* DIRECTION COSINES of A LINE




$\cos\alpha, \cos\beta, \cos\gamma$ are called direction cosines.

$\langle l, m, n \rangle = \langle \cos\alpha, \cos\beta, \cos\gamma \rangle$




$\cos\alpha = \dfrac{x}{\sqrt{x^2+y^2+z^2}}$, $\cos\beta = \dfrac{y}{\sqrt{x^2+y^2+z^2}}$

$\cos\gamma = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$

Ex. $(1, 2, 2)$. $l = \frac{1}{3}$, $m = \frac{2}{3}$, $n = \frac{2}{3}$.

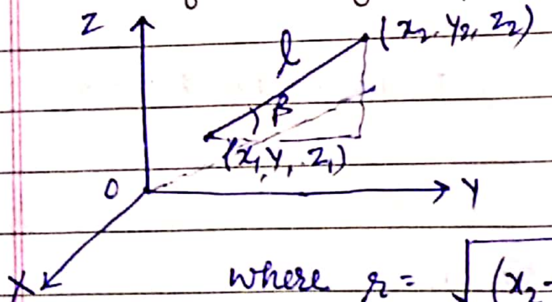
*. $0 \leq \alpha, \beta, \gamma \leq \pi$.

*. Supplements of $\alpha, \beta, \gamma$ are also direction cosines.

so, $\langle -\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3} \rangle$ are also d.c.(s).

Ex. $(1, -2, -5)$ $l = \dfrac{1}{\sqrt{30}}$ $m = \dfrac{-2}{\sqrt{30}}$ $n = \dfrac{-5}{\sqrt{30}}$ | or $\dfrac{-1}{\sqrt{30}}, \dfrac{2}{\sqrt{30}}, \dfrac{5}{\sqrt{30}}$

* D.C. of line joining two points.




$\cos\alpha = \dfrac{x_2 - x_1}{r}$ $\cos\beta = \dfrac{y_2 - y_1}{r}$

$\cos\gamma = z_2 - z_1 / r$

where $r = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$.

Ex. $(2, 1, 6)$ and $(-1, 0, 4)$

$r = \sqrt{9+1+4} = \sqrt{14}$

$\cos\alpha = \dfrac{3}{\sqrt{14}}$ $\cos\beta = \dfrac{1}{\sqrt{14}}$ $\cos\gamma = \dfrac{2}{\sqrt{14}}$

* Properties of Direction Cosines.

(1) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. $\Rightarrow l^2 + m^2 + n^2 = 1$

(2) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

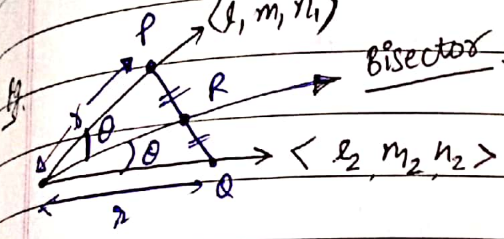
(3) D.C. of line equally inclined Axis

$l^2 = m^2 = n^2$ so, $\boxed{l = m = n = \pm\frac{1}{\sqrt{3}}}$

These are 8 such lines

(4) D.C. of Axes

x axis $(\pm1, 0, 0)$, y $(0, \pm1, 0)$ z $(0, 0, \pm1)$




Eg. find D.C. of bisector.

$P \equiv (rl_1, rm_1, rn_1)$

$Q \equiv (rl_2, rm_2, rn_2)$

R is midpoint of PQ.

$\Rightarrow R \equiv \left(\frac{r}{2}(l_1 + l_2), \frac{r}{2}(m_1 + m_2), \frac{r}{2}(n_1 + n_2)\right)$.

D.C. of bisectors are $\propto [l_1 + l_2, m_1 + m_2, n_1 + n_2]$

Eg. D.C. of 2 lines are $3m + n + 5l = 0$ and $6nl - 2ml + 5nm = 0$

$-n = 3m + 5l \Rightarrow 6l(-5l - 3m) - 2ml + 5(-3m - 5l)m = 0$

$\Rightarrow \cancel{30l}\ 2l^2 + 3lm + m^2 = 0$

$\Rightarrow 2l^2 + 2lm + lm + m^2 = 0 \Rightarrow 2l(l+m) + m(l+m) = 0$

$\therefore \frac{l}{m} = -1$ or $\frac{l}{m} = -\frac{1}{2}$

for first line: $\frac{l_1}{m_1} = -\frac{1}{2} \Rightarrow 3 + \frac{n_1}{m_1} + \frac{5l_1}{m_1} = 0 \Rightarrow \frac{n_1}{m_1} = -\frac{1}{2}$

$m_1 = -2l_1$. $n_1 = l_1$.

as $l^2 + m^2 + n^2 = 1 \Rightarrow l_1^2 + 4l_1^2 + l_1^2 = 1 \Rightarrow \boxed{l_1^2 = \frac{1}{6}}$

$l_1 = \pm\frac{1}{\sqrt{6}}$ . D.C $= \left\langle \pm\frac{1}{\sqrt{6}}, \mp\frac{2}{\sqrt{6}}, \pm\frac{1}{\sqrt{6}} \right\rangle$

Second line $l_2 = -m_2 \Rightarrow 6n_2l_2 + 2l_2^2 - 5n_2l_2 = 0$

$\Rightarrow 2l_2^2 + n_2l_2 = 0 \Rightarrow \cancel{l_2 = -n_2}\ l_2 = -\frac{n_2}{2} \Rightarrow n_2 = -2l_2$

$l_2^2 + l_2^2 + 4l_2^2 = 1 \Rightarrow l_2 = \pm\frac{1}{\sqrt{6}}$

D.C $= \left\langle \pm\frac{1}{\sqrt{6}}, \mp\frac{1}{\sqrt{6}}, \mp\frac{2}{\sqrt{6}} \right\rangle$

## * DIRECTION RATIOS of A LINE

DC(s) $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ . $(l, m, n)$

Ratios $1, -1, 1$ or $2, -2, 2$, or $4, -4, 4$ or

There are infinite Direction Ratios $(a, b, c)$.

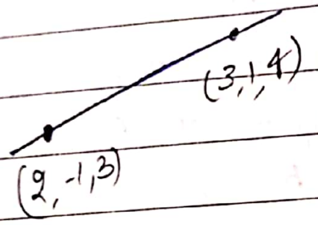
→ $(2,1,3)$, $(4,5,6)$

D.C. $\pm\frac{2}{\sqrt{29}}, \pm\frac{4}{\sqrt{29}}, \pm\frac{3}{\sqrt{29}}$

DR(s) $2, 4, 3$, $-2, -4, -3$, $8, 16, 12$

generally $(a, b, c = \lambda l, \lambda m, \lambda n)$

Eg.

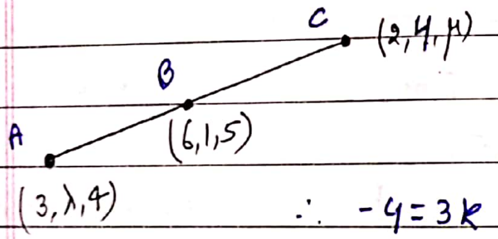



D.R. = $(1, 2, 1)$.
$(\hat{i}+2\hat{j}+\hat{k})$ is $\parallel$ to line.

D.C = $\pm\left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

$\pm\frac{1}{\sqrt{6}}(\hat{i}+2\hat{j}+\hat{k})$ is a unit vector $\parallel$ to line.

Ex.




find λ & μ for collinearity

D.R of AB $\propto$ BC.

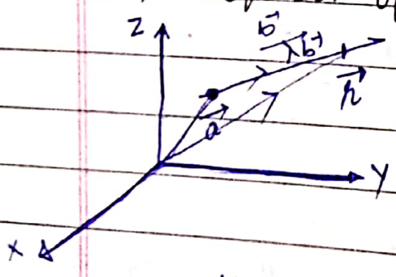
$(-4, 3, \mu-5) = k(3, 1-\lambda, 1)$

$\therefore -4 = 3k \Rightarrow k = -4/3$

$\frac{-4}{3} = \frac{3}{1-\lambda} = \frac{\mu-5}{1} \Rightarrow -4+4\lambda = 9 \Rightarrow \boxed{\lambda = 13/4}$

$3\mu - 15 = -4 \Rightarrow \boxed{\mu = 11/3}$

## * Vector Equation of a line



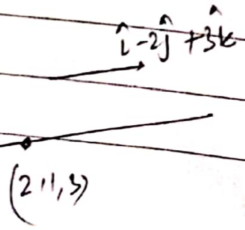

$\vec{r}$ = (general point) $\vec{a}$ = (point on line)

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\boxed{\vec{r} = \vec{a} + \lambda\hat{b}}$

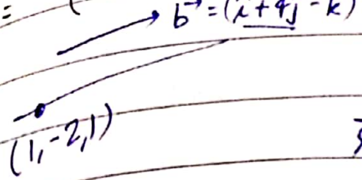
This is the vector Eqn. of line passing through $\vec{a}$ and parallel to $\vec{b}$.

Eg.




$\vec{r} = 2\hat{i}+\hat{j}+3\hat{k} + \lambda(\hat{i}-2\hat{j}+3\hat{k})$

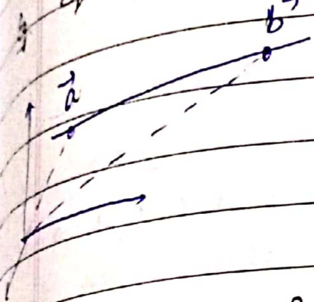
Q. $\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(\hat{i} + 4\hat{j} - \hat{k})$ find Eq$^n$ of parallel line through $(-3, 1, 6)$.




D.R = $1, 4, -1$

$\vec{r} = (-3\hat{i} + \hat{j} + 6\hat{k}) + \lambda(\hat{i} + 4\hat{j} - \hat{k})$

Q. Eq$^n$ of line passing through Two points




$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

Q. A line through $(2, 1, 4)$ and $(-1, 3, 6)$

$\vec{r} = (2\hat{i} + \hat{j} + 4\hat{k}) + \lambda(-3\hat{i} + 2\hat{j} + 2\hat{k})$

* Cartesian Equation of a Line:

$\vec{r} = \vec{a} + \lambda\vec{b}$

eg. $(x\hat{i} + y\hat{j} + z\hat{k}) = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 4\hat{j} + 2\hat{k})$.

Equating: $x = 2 + \lambda$, $y = -1 - 4\lambda$, $z = 3 + 2\lambda$

$\lambda = x - 2$ $\quad \lambda = \frac{-1-y}{4}$ $\quad \lambda = \frac{z-3}{2}$

So, $\boxed{\frac{x-2}{1} = \frac{y+1}{-4} = \frac{z-3}{2} = \lambda}$

In generall, $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \lambda$.

$a, b, c$ are D.R(s); $(x_1, y_1, z_1)$ is a point on line.

Q. write vector Eq$^n$ of $\frac{x-3}{2} = \frac{y-4}{0} = \frac{z+5}{1}$.

$\vec{r} = (3\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(2\hat{i} + \hat{k})$

Q. $2x - 1 = 3y + 2 = -6z + 12$, find D.R(s), vector form.

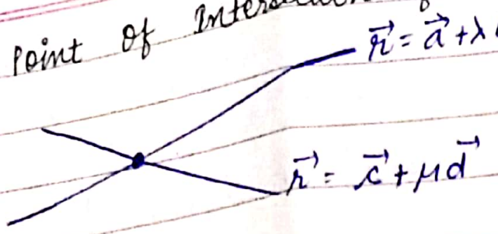
$\frac{x - 1/2}{1/2} = \frac{y + 2/3}{1/3} = \frac{z - 2}{-1/6} \Rightarrow \frac{x - 1/2}{3} = \frac{y + 2/3}{2} = \frac{z - 2}{-1}$

D.R: $(3, 2, -1)$, DC $\pm\left(\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$

$\vec{r} = \left(\frac{1}{2}\hat{i} - \frac{2}{3}\hat{j} + 2\hat{k}\right) + \lambda(3\hat{i} + 2\hat{j} - \hat{k})$

Scanned with CamScanner

* Point of Intersection of Lines




At intersection point,
$\vec{a} + \lambda\vec{b} = \vec{c} + \mu\vec{d}$

$$(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$$
$$= (c_1\hat{i} + c_2\hat{j} + c_3\hat{k}) + \mu(d_1\hat{i} + d_2\hat{j} + d_3\hat{k})$$

$\Rightarrow$ $\left.\begin{aligned} a_1 + \lambda b_1 &= c_1 + \mu d_1 \\ a_2 + \lambda b_2 &= c_2 + \mu d_2 \\ a_3 + \lambda b_3 &= c_3 + \mu d_3 \end{aligned}\right\}$ find $\lambda$ and $\mu$ from any 2 eq$^n$s. If $\lambda$ and $\mu$ satisfy the 3rd equation. then the **lines intersect**, otherwise Not.

→ only coplanar lines intersect.

→ $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} = \lambda \Rightarrow x = a_1\lambda + x_1 \quad y = b_1\lambda + y_1 \quad z = c_1\lambda + z_1$

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2} = \mu \Rightarrow x = a_2\mu + x_2 \quad y = b_2\mu + y_2$

~~$z = z_2\lambda + z_2$~~

$z = c_2\lambda + z_2$

Equate $x, y, z$ and find $\lambda$ and $\mu$.

Eg $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$ ; $\frac{x-4}{5} = \frac{y-1}{2} = z = \mu$

$x = 2\lambda + 1 \quad y = 3\lambda + 2 \quad z = 4\lambda + 3$ | $x = 5\mu + 4 \quad y = 2\mu + 1 \quad z = \mu$

$\mu = 4\lambda + 3$ ; $2\mu + 1 = 3\lambda + 2$

$2(4\lambda + 3) + 1 = 3\lambda + 2 \Rightarrow 8\lambda + 7 = 3\lambda + 2 \Rightarrow 5\lambda = -5 \Rightarrow \lambda = -1$

$\mu = -1$ ✓

$2\lambda + 1 = -1$, $5\mu + 4 = -1 \Rightarrow$ satisfy $\Rightarrow$ intersecting

point of intersection $= (-1, -1, -1)$ ✓

Eg. $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} + m\hat{k})$ ; $\vec{r} = (2\hat{i} + \hat{j} + 2\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$.

$1 + \lambda = 2 + \mu$ ; $2 + \lambda = 1 + 2\mu$, $1 + \lambda m = 2 + 2\mu$.

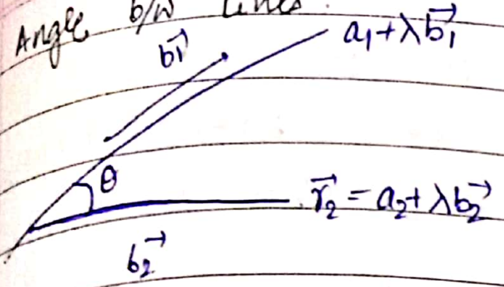
$\lambda = 1 + \mu \Rightarrow 3 + \mu = 1 + 2\mu$

$\Rightarrow \boxed{\mu = 2} \boxed{\lambda = 3}$

so, $1 + 3m = 2 + 4$

$3m = 5$ $\boxed{m = 5/3}$

* Angle b/w Lines.




$$\cos\theta=\frac{\vec{b_1}\cdot\vec{b_2}}{|\vec{b_1}|\cdot|\vec{b_2}|}$$

* $\dfrac{x-x_1}{a_1}=\dfrac{y-y_1}{b_1}=\dfrac{z-z_1}{c_1}$ | $a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$.

$\dfrac{x-x_2}{a_2}=\dfrac{y-y_2}{b_2}=\dfrac{z-z_2}{c_2}$ | $a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$.

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

if D.C. are given, $\cos\theta=l_1l_2+m_1m_2+n_1n_2$.

Q $\dfrac{x+1}{3}=\dfrac{y-6}{2}=\dfrac{z-3}{6}$, $\vec{r}=\hat{i}-2\hat{j}+3\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$.

$3\hat{i}+2\hat{j}+6\hat{k}$.

$$\cos\theta=\frac{(3\hat{i}+2\hat{j}+6\hat{k})(\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{49}\cdot\sqrt{9}}=\frac{3+4+12}{21}=\frac{19}{21}$$

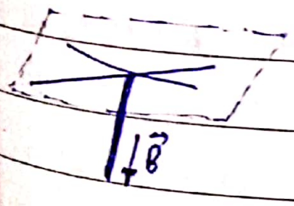
$\therefore$ $\boxed{\theta=\cos^{-1}(19/21)}$

* for $\perp$r lines, $a_1a_2+b_1b_2+c_1c_2=0$
or $l_1l_2+m_1m_2+n_1n_2=0$.

* for $\parallel$ lines $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}$

Q $\vec{r}=\hat{i}+2\hat{j}-\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})\rightarrow$ line I.

D.R of line II. $-2,-2,1$. find the line passing through $(2,1,6)$ and $\perp$r to both given lines.



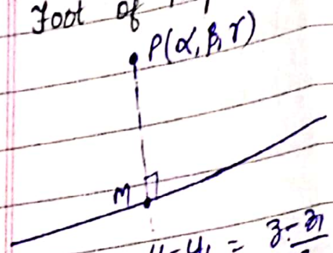

It is cross product of $\vec{b_1}$ and $\vec{b_2}$.

$$\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&2\\-2&-2&1\end{vmatrix}=6\hat{i}-5\hat{j}+2\hat{k}.$$

$\vec{r}=(2\hat{i}+\hat{j}+6\hat{k})+\lambda(6\hat{i}-5\hat{j}+2\hat{k})$ Ans.

Scanned with CamScanner

* **Foot of Perpendicular**




M is foot of $\perp r$.

$M = (a\lambda + x_1,\ b\lambda + y_1,\ c\lambda + z_1)$.

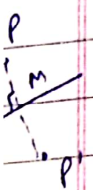
as $\perp r$ lines, $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \lambda$ | $DR(PM) = (\alpha - a\lambda - x_1,\ \beta - b\lambda - y_1,\ \gamma - c\lambda - z_1)$

$\perp r$ $a(\alpha - a\lambda - x_1) + b(\beta - b\lambda - y_1) + c(\gamma - c\lambda - z_1) = 0$

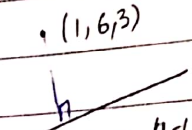
find $\lambda$ and hence $M$.

* $\perp r$ distance b/w line and point = PM




* P' is the image of P wrt line.
M is midpoint of PP', we can find P'.

Eg. 




$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

$x = \lambda \quad y = 2\lambda + 1 \quad z = 3\lambda + 2$

D.R. $(\lambda - 1)\ (2\lambda - 5),\ (3\lambda - 1)$

$\therefore 1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$

$\Rightarrow 14\lambda - 14 = 0 \Rightarrow \lambda = 1$.

$M = (1, 3, 5)$

$\perp r$ distance $PM = \sqrt{3^2 + 2^2} = \sqrt{13}$.

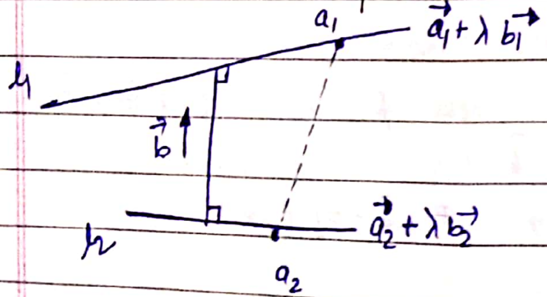
$\hat{i} + 6\hat{j} + 3\hat{k} + x\hat{i} + y\hat{j} + z\hat{k} = 2\hat{i} + 6\hat{j} + 10\hat{k}$

$\Rightarrow x = 1 \quad y = 0, \quad z = 7$ $\therefore$ image $= \boxed{\hat{i} + 7\hat{k}}$

* **SKEW LINES** -

~~Non-plane~~ Non parallel & non intersecting lines.

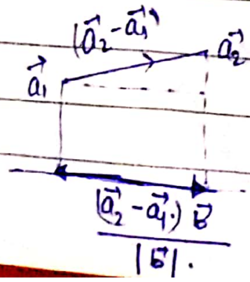
→ Shortest distance b/w skew lines.




$\vec{b} = \vec{b_1} \times \vec{b_2}$

shortest distance is the projection of $(\vec{a_2} - \vec{a_1})$ on $\vec{b}$

Shortest distance $= \frac{(\vec{a_1} - \vec{a_2}) \cdot \vec{b}}{|\vec{b}|}$




$= \frac{(\vec{a_1} - \vec{a_2}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}$

$= \frac{[\vec{a_1} - \vec{a_2} \quad \vec{b_1} \quad \vec{b_2}]}{|\vec{b_1} \times \vec{b_2}|}$

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ ; $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$

$\vec{b_1} = 2\hat{i}, 3\hat{j}, 4\hat{k}$ . $\vec{b_2} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ .

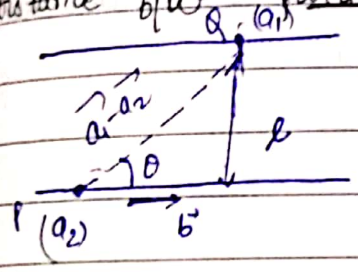
$a_1 = \hat{i} + 2\hat{j} + 3\hat{k}$, $a_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$ $\} \Rightarrow \hat{i} + \hat{j} + \hat{k}$

$[\vec{a_2} - \vec{a_1}\ \vec{b_1}\ \vec{b_2}] = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 1(-1) - 1(-2) + 1(-1) = \underline{0}$ ✓

$|\vec{b_1} \times \vec{b_2}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = |-\hat{i} + 2\hat{j} - \hat{k}| = \sqrt{6}$ ✓

S.D $= 0 \Rightarrow$ lines are intersecting they aren't skew.

* Distance b/w Parallel lines



$\vec{r} = a_1 + \lambda \vec{b}$

$\vec{r} = a_2 + \mu \vec{b}$

$\vec{b} \times (\vec{a_1} - \vec{a_2}) = |\vec{b}||\vec{a_1} - \vec{a_2}| \sin\theta\, \hat{n}$

$\therefore |\vec{b} \times (\vec{a_1} - \vec{a_2})| = |\vec{b}| PQ \sin\theta$

$= |\vec{b}| \cdot l$

so, $l = \frac{|\vec{b} \times (\vec{a_1} - \vec{a_2})|}{|\vec{b}|}$

Q. $\vec{r} = \hat{i} + \hat{j} + 3\hat{k} + \lambda(2\hat{i} + 2\hat{j} + \hat{k})$

$\frac{x-3}{2} = \frac{y-1}{2} = \frac{z-1}{1}$

$\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$ .

$a_1 = \hat{i} + \hat{j} + 3\hat{k}$ $\vec{a_2} = 3\hat{i} + \hat{j} + \hat{k}$ .

$|\vec{b}| = 3$ .

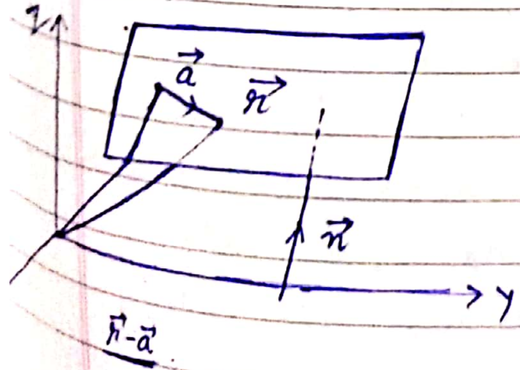
$\vec{a_1} - \vec{a_2} = -2\hat{i} + 2\hat{k}$ .

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ -2 & 0 & 2 \end{vmatrix} = \hat{i}(4) - \hat{j}(6) + \hat{k}(-4) = 4\hat{i} - 6\hat{j} - 4\hat{k}$

$\therefore |\vec{b} \times (\vec{a_1} - \vec{a_2})| = \sqrt{68} = 2\sqrt{17}$

so, ⊥r distance $= \frac{2}{3}\sqrt{17}$ ✓

* PLANE




$\vec{r}$ = general point in plane

✓ $\vec{a}$ = a specific point in plane.

$\vec{r} - \vec{a}$ always ⊥r to $\vec{n}$.

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

✓ $\vec{N}$ = A vector normal to plane

(2 things needed).

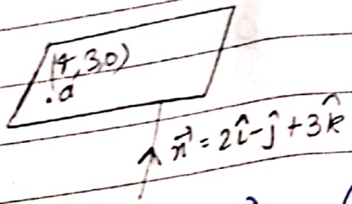
$$\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n} = d$$

$$\therefore (x\hat{i}+y\hat{j}+z\hat{k})\cdot(a\hat{i}+b\hat{j}+c\hat{k}) = d$$

$$\Rightarrow \boxed{ax+by+cz=d}$$

where a, b, c are direction ratios of normal to plane.

Eg




$$\vec{a}\cdot\vec{n} = (4\hat{i}+3\hat{j}+0\hat{k})(2\hat{i}-\hat{j}+3\hat{k})$$
$$= 8-3 = 5.$$

$$(x\hat{i}+y\hat{j}+z\hat{k})(2\hat{i}-\hat{j}+3\hat{k})=5$$

$$\Rightarrow 2x-y+3z=5$$

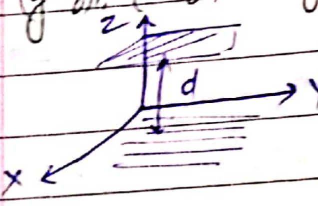
$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$$

where $(x_1, y_1, z_1) = \vec{a}$ and a, b, c are direction ratios.

Eg. $2(x-4)+(-1)(y-3)+3(z-0)=0$

$$2x-y+3z = 8-3 = 5$$

Ex (y-z), (x-z), (x-y) planes.




for (x,y) plane, $\vec{n}=\hat{k}$, $\vec{a}=(0,0,0)$

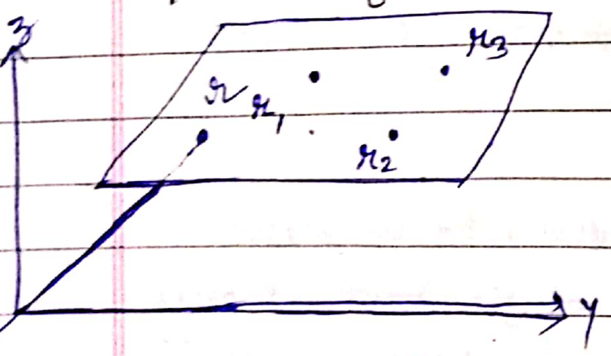
$\therefore \boxed{z=0}$

y-z, $\boxed{x=0}$, x-z $\boxed{y=0}$

A plane $\parallel$ to x-y plane, d apart from (0,0,0)

$\boxed{z=d}$

* Equation of plane passing through three non collinear pts.




$$\vec{n} = (\vec{r}_2-\vec{r}_1)\times(\vec{r}_2-\vec{r}_3).$$

$$\Rightarrow \vec{n} = \vec{r}_2\times\vec{r}_1 - \vec{r}_3\times\vec{r}_1 + \vec{r}_3\times\vec{r}_2$$

$$(\vec{r}-\vec{r}_1)\cdot\vec{n} = 0$$

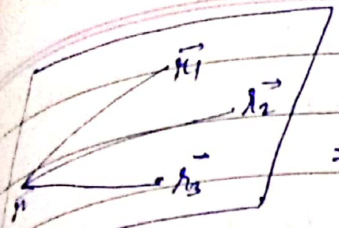
$$\Rightarrow \vec{n}\cdot\vec{r} = \vec{r}_1\cdot\vec{n}$$

$$\Rightarrow \vec{r}\cdot(\vec{r}_2\times\vec{r}_1+\vec{r}_1\times\vec{r}_3+\vec{r}_3\times\vec{r}_2)$$
$$= \vec{r}_1\cdot(\vec{r}_2\times\vec{r}_1+\vec{r}_1\times\vec{r}_3+\vec{r}_3\times\vec{r}_2).$$

$$\Rightarrow [\vec{r}\ \vec{r}_2\ \vec{r}_1]+[\vec{r}\ \vec{r}_1\ \vec{r}_3]+[\vec{r}\ \vec{r}_3\ \vec{r}_2] = [\vec{r}_1\ \vec{r}_3\ \vec{r}_2]$$

$$\Rightarrow [\vec{r}_1\ \vec{r}_2\ \vec{r}]+[\vec{r}_1\ \vec{r}\ \vec{r}_3]+[\vec{r}\ \vec{r}_2\ \vec{r}_3] = [\vec{r}_1\ \vec{r}_3\ \vec{r}_2]$$

Scanned with CamScanner



The three vectors are coplanar.

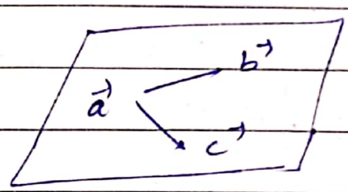
$[\vec{r}-\vec{r_1} \quad \vec{r}-\vec{r_2} \quad \vec{r}-\vec{r_3}] = 0$

$$\Rightarrow \begin{vmatrix} x-x_1 & x-x_2 & x-x_3 \\ y-y_1 & y-y_2 & y-y_3 \\ z-z_1 & z-z_2 & z-z_3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-x_1 & x_2-x_3 & x_3-x_1 \\ y-y_1 & y_2-y_3 & y_3-y_1 \\ z-z_1 & z_2-z_3 & z_3-z_1 \end{vmatrix} = 0$$

Q. $(2,1,3)$, $(1,4,0)$, $(5,2,1)$

$$\begin{vmatrix} x-2 & x-1 & x-5 \\ y-1 & y-4 & y-2 \\ z-5 & z & z-1 \end{vmatrix} = 0$$



$\vec{a}-\vec{b} = \hat{i}-3\hat{j}+3\hat{k}$ , $\vec{c}-\vec{a} = 3\hat{i}+\hat{j}-2\hat{k}$

$\vec{n} = (\hat{i}-3\hat{j}+3\hat{k}) \times (3\hat{i}+\hat{j}-2\hat{k})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 3 \\ 3 & 1 & -2 \end{vmatrix} = 3\hat{i}+11\hat{j}+10\hat{k}$$

$\Rightarrow 3(x-1)+11(y-4)+10(z) = 0$

$\boxed{3x+11y+10z = 47}$

* Coplanarity of four points

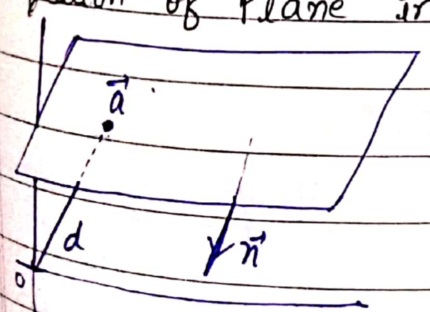
$[\vec{a}-\vec{b} \quad \vec{b}-\vec{c} \quad \vec{c}-\vec{d}] = 0$

Q. find eq$^n$ of plane through 3 points &

$$\begin{vmatrix} x-x_1 & x-x_2 & x-x_3 \\ y-y_1 & y-y_2 & y-y_3 \\ z-z_1 & z-z_2 & z-z_3 \end{vmatrix} = 0$$

if $x_4, y_4, z_4$ satisfy this eq$^n$ then the 4 points are coplanar.

* Equation of Plane in Normal form.



d is ⊥r distance of plane from origin.

d is parallel to $\vec{n}$.

so, $\vec{a} = d\hat{n}$

$(\vec{r}-\vec{a})\cdot\vec{n} = 0$

$\Rightarrow \vec{r}\cdot\vec{n} = d\,\hat{n}\cdot\vec{n} \Rightarrow \vec{r}\cdot\vec{n} = |\vec{n}|\,d.$

$\Rightarrow \boxed{\vec{r}\cdot\hat{n} = d}$ (Normal form)

Eg $\vec{n} = 2\hat{i}+\hat{j}+2\hat{k}$ $\quad d = 12.$

$\therefore \boxed{\frac{2x+y+2z}{3}=12}$ or $2x+y+2z=36$

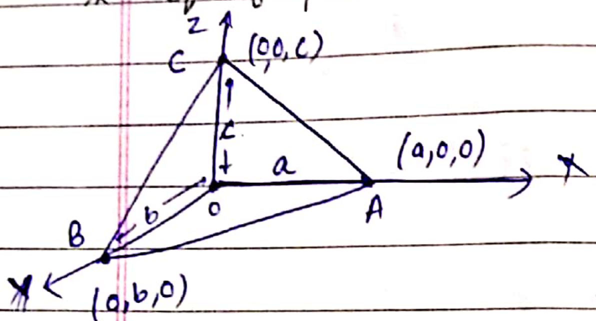
Eg $x-y+4z=10$ find d.

D.R of $\vec{n}$ : $(1,-1,4)$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}-\hat{j}+4\hat{k}) = 10$

$\Rightarrow \frac{\vec{r}\cdot\vec{n}}{\sqrt{18}} = \frac{10}{\sqrt{18}} \Rightarrow \vec{r}\cdot\hat{n} = d$ where $\boxed{d = \frac{10}{\sqrt{18}}}$

* Eq. of plane in Intercept form.

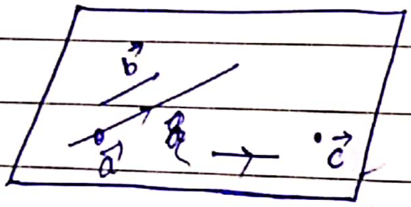



$A(a,0,0)$. $B(0,b,0)$. $C(0,0,c)$

$$\begin{vmatrix} x-a & 0 & -a \\ y-0 & b & 0 \\ z-0 & -c & c \end{vmatrix} = 0$$

$\Rightarrow \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

* Equation of a plane containing a given line




line: $\vec{r} = \vec{a}+\lambda\vec{b}$

$(\vec{r}-\vec{a})\cdot\vec{n}=0$

$(\vec{c}-\vec{a})\times\vec{b} = \vec{n}$

$\Rightarrow (\vec{r}-\vec{a})\cdot((\vec{c}-\vec{a})\times\vec{b}) = 0$

Eg Eq$^n$ of a plane containing $\vec{r} = (\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ and passing through $(3,2,1)$

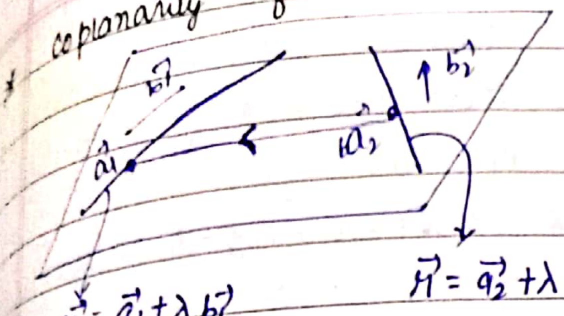
$\vec{c}-\vec{a} = 2\hat{i}-2\hat{k}$ ; $\quad \vec{b} = 2\hat{i}+3\hat{j}+4\hat{k}$

~~K-a~~ $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -2 \\ 2 & 3 & 4 \end{vmatrix} = 6\hat{i}-12\hat{j}+6\hat{k}.$

$((x-1)\hat{i}+(y-2)\hat{j}+(z-3)\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k})=0$

$\Rightarrow x-\cancel{1}-2y+\cancel{4}+z-\cancel{3} = 0 \Rightarrow \boxed{x-2y+z=0}$ (passing through origin)

* coplanarity of Two lines




$$[\vec{a_1}-\vec{a_2} \;\; \vec{b_1} \;\; \vec{b_2}] = 0$$

$\vec{r_1} = \vec{a_1} + \lambda \vec{b_1}$ $\qquad$ $\vec{r} = \vec{a_2} + \lambda \vec{b_2}$.

Q. $\frac{x-1}{2} = \frac{y}{1} = \frac{z-1}{2}$ ; $\frac{x}{3} = \frac{y-\lambda}{1} = \frac{z}{0}$ coplanar

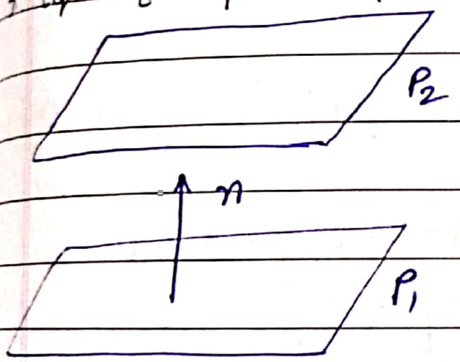
$\vec{a_1} = (1,0,1)$ $\quad$ $\vec{a_2} = (0,\lambda,0)$ $\quad$ $\vec{b_1} = 2\hat{i}+\hat{j}+2\hat{k}$ $\quad$ $\vec{b_2} = 3\hat{i}+\hat{j}$.

$\vec{a_1} - \vec{a_2} = \hat{i} - \lambda\hat{j} + \hat{k}$

$$\begin{vmatrix} 1 & -\lambda & 1 \\ 2 & 1 & 2 \\ 3 & 1 & 0 \end{vmatrix} = 1(-2) + \lambda(-6) + 1(-1) = 0$$

$\Rightarrow 6\lambda = -3 \Rightarrow \boxed{\lambda = -\frac{1}{2}}$

* Eq$^n$ of plane parallel to a plane -




Normal vector is same.

$\vec{r}\cdot\vec{n_1} = d_1 \Rightarrow ax+by+cz = d$

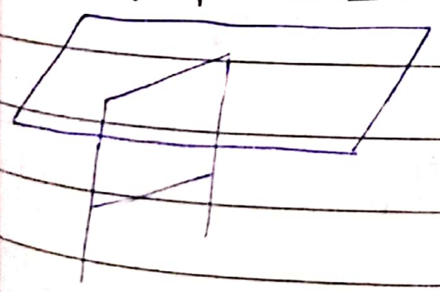
$\boxed{ax+by+cz = \lambda}$ is the req. eq$^n$ of parallel plane.

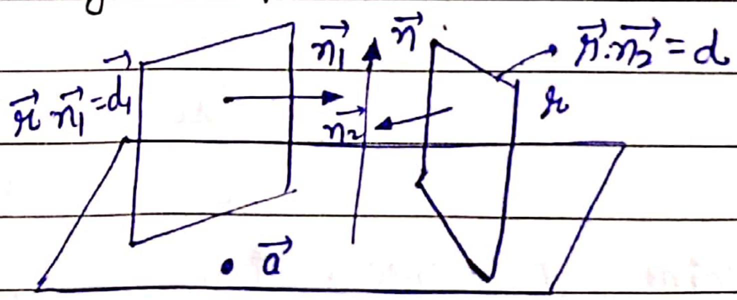
Q. find plane $\parallel$ to $2x-y+3z = 4$ and passing through $(1,4,3)$.

$2x-y+3z = \lambda$ $\qquad \Rightarrow$ $2x-y+3z=7$

$\therefore 2-4+9 = \lambda = 7$

* Eq$^n$ of plane $\perp$r to given plane.

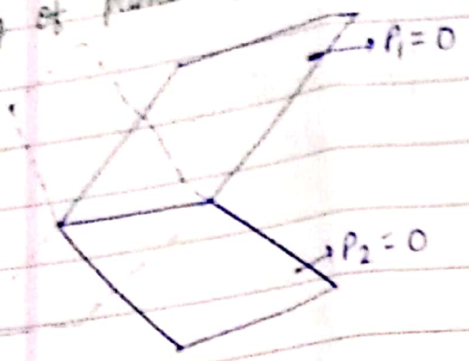





$\vec{n_1} \times \vec{n_2} = \vec{n}$

$(\vec{r}-\vec{a})\vec{n} = 0$

* Eq of plane passing through given planes




Infinite planes can pass through line of intersection.

$P_1 + \lambda P_2 = 0$.

$\Rightarrow (a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$ ✓

Eg $2x+y+z+1=0$, $x+y+z+2=0$ find a plane

(1) passing through intersection and $(2,1,0)$

$(2x+y+z+1) + \lambda(x+y+z+2) = 0$

$\Rightarrow (4+1+1) + \lambda(2+1+2) = 0 \Rightarrow 6+5\lambda = 0 \Rightarrow \lambda = -6/5$.

$\therefore \Rightarrow \frac{4}{5}x - \frac{1}{5}y - \frac{1}{5}z - \frac{7}{5} = 0 \Rightarrow \boxed{4x - y - z = 7}$

(2) $2x-y+z+1=0$ $x+y+z+2=0$ find a ⊥r plane passing to $x+y-3z=0$ through line of intersection :

$(2x-y+z+1) + \lambda(x+y+z+2)$ | $\vec{n} = (2+\lambda)\hat{i} + (\lambda-1)\hat{j} + (1+\lambda)\hat{k}$.

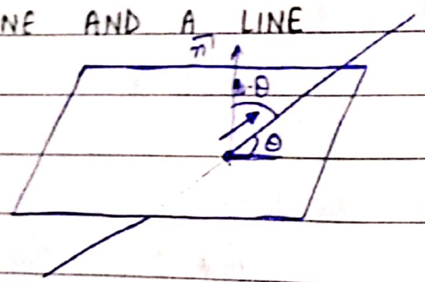
$(2+\lambda)1 + (\lambda-1)1 - 3(1+\lambda) = 0$

$\Rightarrow 2+\lambda+\lambda-1-3\lambda-3 = 0 \Rightarrow -\lambda-2=0 \Rightarrow \boxed{\lambda=-2}$ ✓

Plane : $-3y - z - 3 = 0 \Rightarrow \boxed{3y+z=3}$ ✓

* PLANE AND A LINE

✓ Angle




$\vec{r} = \vec{a} + \lambda\vec{b}$ (line)

$\vec{r} \cdot \vec{n} = d$ (plane)

$\cos(90-\theta) = \dfrac{\vec{n}\cdot\vec{b}}{|\vec{n}||\vec{b}|}$

$\Rightarrow \boxed{\sin\theta = \dfrac{\vec{n}\cdot\vec{b}}{|\vec{n}||\vec{b}|}}$ ✓

✓ Point of Intersection

$\Rightarrow (\vec{a}+\lambda\vec{b})\cdot\vec{n} = d \Rightarrow \boxed{\lambda = \dfrac{d - \vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}}$

eg. $\dfrac{x-2}{3} = \dfrac{y-3}{4} = \dfrac{z-4}{5}$, $2x-2y+z=5$.

$\vec{n} = 2\hat{i} - 2\hat{j} + \hat{k}$ $\quad \vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ $\quad \vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ $\quad d = 5$

$$\sin\theta = \frac{6 - 8 + 5}{3 \cdot 5\sqrt{2}} = \frac{1}{5\sqrt{2}} \Rightarrow \boxed{\theta = \sin^{-1}\left(\frac{1}{5\sqrt{2}}\right)}$$

Another Angle $= (\pi - \theta)$

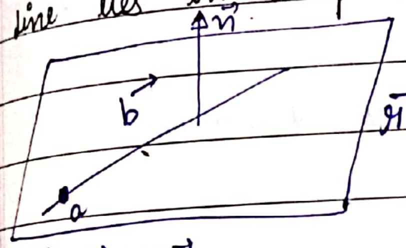
$x = 3\lambda + 2$, $\quad y = 4\lambda + 3$, $\quad z = 5\lambda + 4$

$\therefore 2(3\lambda + 2) - 2(4\lambda + 3) + 5\lambda + 4 = 5$

$\Rightarrow 3\lambda + 2 = 5 \Rightarrow \underline{\lambda = 1}$

point $= (5, 7, 9)$

# Line lies in a plane



$$\boxed{\begin{array}{c} \vec{a} \cdot \vec{n} = d \\ \vec{b} \cdot \vec{n} = 0 \end{array}}$$ (a is satisfying).

$\vec{r} \cdot \vec{n} = d$,

$\vec{r} = \vec{a} + \lambda\vec{b}$

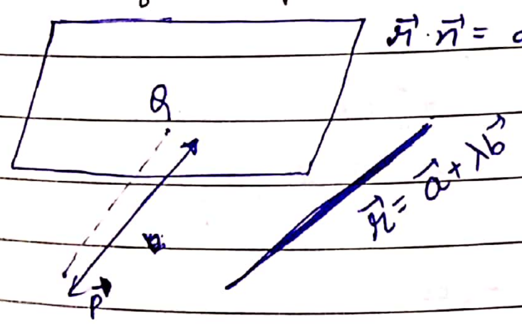
Ex $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ ; $\ell x - 4y + z = 5$. if line lies in a plane ; find $\ell$ & $k$.

$(4, 2, k) \Rightarrow 4\ell - 8 + k = 5 \Rightarrow \cancel{\ell = 3}$ $\quad 4\ell + k = 13$ — (1)

$\vec{b} \cdot \vec{n} = 0 \Rightarrow (\hat{i} + \hat{j} + 2\hat{k})(\ell\hat{i} - 4\hat{j} + 1\hat{k}) = 0$

$\Rightarrow \ell - 4 + 2 = 0 \Rightarrow \boxed{\ell = 2}$ $k = 5$

# Distance of a point from a plane along a line.



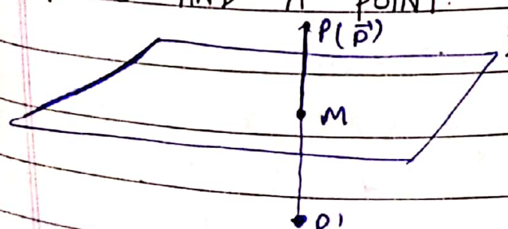
$\vec{r} \cdot \vec{n} = d$

Eqn of line
$\vec{r} = \vec{p} + \mu\vec{b}$

find point of intersecti of line with Plane (Q).

find distance PQ.

## * PLANE AND A POINT



$\vec{r} \cdot \vec{n} = d$

M is foot of $\perp$r,
P' is image of P w.r.t. plane.

Eqn of line PM $\Rightarrow \vec{p} + \lambda\vec{n} = \vec{r}$

$\Rightarrow (\vec{p} + \lambda\vec{n}) \cdot \vec{n} = d$

find $\lambda$, and $\vec{r}$ ie, M.

Ex $2x+y+z=0$ $\quad (3,5,7)$ $\quad = (3+2\lambda)\hat{i}+(5+\lambda)\hat{j}+(7+\lambda)\hat{k}$

$\vec{r} = (3\hat{i}+5\hat{j}+7\hat{k}) + \lambda(2\hat{i}+\hat{j}+\hat{k})$

$\vec{r}\cdot\vec{n} = d \Rightarrow ((3+2\lambda)\hat{i}+(5+\lambda)\hat{j}+(7+\lambda)\hat{k})\cdot(2\hat{i}+\hat{j}+\hat{k}) = 0$

$\Rightarrow 6+4\lambda+5+\lambda+7+\lambda = 0$

$\Rightarrow 6\lambda = -18 \Rightarrow \boxed{\lambda = -3}$

foot of ⊥r $= \boxed{-3\hat{i}+2\hat{j}+4\hat{k}}$ $\quad (-3,2,4)$

for Image. $(3\hat{i}+5\hat{j}+7\hat{k}) + \vec{I}' = 2(-3\hat{i}+2\hat{j}+4\hat{k})$

$\vec{I}' = \boxed{-9\hat{i}-\hat{j}+\hat{k}}$

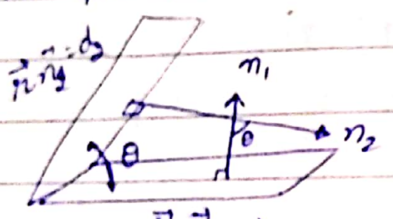
⊥r distance $= \sqrt{6^2+3^2+3^2} = \sqrt{36+18} = \sqrt{54} = 3\sqrt{6}$

- $ax+by+cz+d=0$ $\quad (x,y,z)$ $\quad$ find ⊥r distance.

$$\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

## * PLANE AND A PLANE

- Angle b/w Planes.




$$\cos\theta = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$$

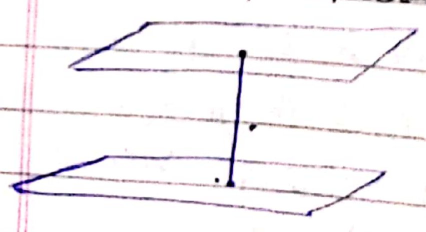
Ex $x-y+z=4$ $\qquad x+y+z=6$

$n_1 = \hat{i}-\hat{j}+\hat{k}$ $\qquad \vec{n_2} = \hat{i}+\hat{j}+\hat{k}$

$\cos\theta = \dfrac{1-1+1}{\sqrt{3}\cdot\sqrt{3}} = \dfrac{1}{3} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{1}{3}\right)}$

obtuse $= \pi - \cos^{-1}(1/3)$.

- Distance b/w Parallel planes



$ax+by+cz+d_1=0$

$ax+by+cz+d_2=0$

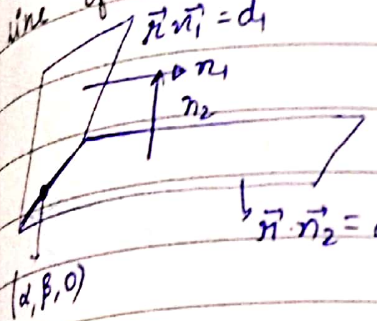
distance $= \left|\dfrac{d_1-d_2}{\sqrt{a^2+b^2+c^2}}\right|$

Scanned with CamScanner

$2x - y + 2z + 4 = 0$ ; $2x - y + 2z - 5 = 0$

$\left|\dfrac{4+5}{\sqrt{4+1+4}}\right| = \dfrac{9}{3} = 3$

Line of Intersection.




$\vec{b} = \vec{n_1} \times \vec{n_2}$

get any point which satisfies both planes.

line: $\vec{r} = (\alpha\hat{i} + \beta\hat{j}) + \lambda(\vec{n_1} \times \vec{n_2})$

$4x + 4y - 5z = 12$ $\qquad 8x + 12y - 13z = 32$

$$\vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & -5 \\ 8 & 12 & -13 \end{vmatrix} = \hat{i}(8) - \hat{j}(-12) + \hat{k}(16) = 8\hat{i} + 12\hat{j} + 16\hat{k}$$

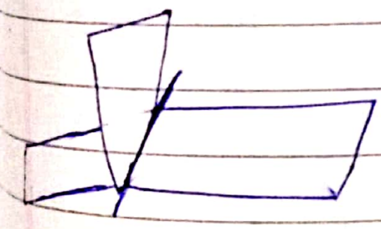
let $z = 0$, $\quad 4x + 4y = 12 \Rightarrow x + y = 3 \Rightarrow 2x + 2y = 6$

$8x + 12y = 32 \Rightarrow 2x + 3y = 8$

$\boxed{y = 2}$ $\boxed{x = 1}$.

$(1,2,0)$ is common pt. $\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 0\hat{k}) + \lambda(8\hat{i} + 12\hat{j} + 16\hat{k})$

$kx + 4y + z = 0$ $\qquad 4x + ky + 2z = 0$ $\qquad 2x + 2y + z = 0$ are meeting in a line. find $k$.



The three equations have infinite no of solutions.

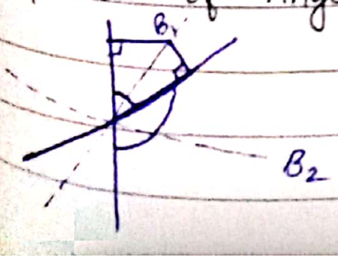
$$\Rightarrow \begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0 \Rightarrow k(k-4) - 4(4-4) + 1(8-2k) = 0$$

$\Rightarrow k^2 - 4k + 8 - 2k = 0$

$\Rightarrow k(k-4) - 2(k-4) = 0$

$\Rightarrow \boxed{k = 4}$ $\boxed{k = 2}$

Equation of Angle Bisectors




$$\left|\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}\right| = \left|\frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$\Rightarrow \frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \pm\left(\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}\right)$$

Working Rule: make $d_1$ and $d_2$ both positive.

→ if $a_1a_2+b_1b_2+c_1c_2>0$

then $(\quad) = -(\quad)$ and $(\quad) = +(\quad)$

Acute Angle obtuse angle

→ if $a_1a_2+b_1b_2+c_1c_2<0$

then $(\quad) = -(\quad)$ and $(\quad) = +(\quad)$

obtuse angle Acute angle

→ if $a_1a_2+b_1b_2+c_1c_2=0$

then planes intersect at right angles.

Ex. $2x-y+2z+3=0$ $\quad a_1a_2+b_1b_2+c_1c_2$

$3x-2y+6z+8=0$ $\quad = 2\times3+(-1)(-2)+2\times6>0$

obtuse $\quad \frac{2x-y+2z+3}{\sqrt{4+1+4}} = \frac{3x-2y+6z+8}{\sqrt{9+4+36}}$

$\Rightarrow 14x-7y+14z+21 = 9x-6y+18z+24$

$\Rightarrow \boxed{5x-y-4z=3}$

Acute $\quad 14x-7y+14z+21 = -9x+6y-18z-24$

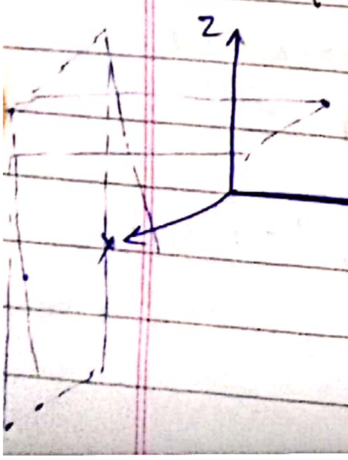
$\Rightarrow \boxed{23x-13y+32z+45=0}$

→ If $a_1a_2+b_1b_2+c_1c_2>0$ then $(+)$ will contain origin. (obtuse)

→ If $a_1a_2+b_1b_2+c_1c_2<0$ then $(+)$ will contain origin (acute)

Exercise 12.1

1. Planes are drawn ∥ to coordinate planes through $(1,2,3)$ and $(3,-4,-5)$. length of edges of ∥opiped →




edges length are

$(3-1), (-4-2), -5-3$

$\Rightarrow$ $2, 6, 8$