Concepts and Formulas

Matrix and Determinant

Symmetric and Skew Symmetric matrices

- **Symmetric Matrix** - If $A^T = A$
- **Skew - symmetric Matrix** - If $A^T = A$
  Note: In a skew matrix, diagonal elements are **always 0** .
- For any square matrix A,
  $(A + A^T)$ is a symmetric matrix
  $(A - A^T)$ is a skew-symmetric matrix

Inverse of a matrix

For a square matrix A, if

$AB = BA = I$

Then, B is the inverse of A

i.e. $\mathbf{B = A^{-1}}$

We will find inverse of a matrix by

- Elementary transformation
- Using adjoint

Properties of Inverse

1. For a matrix A,
   A^{-1} is unique, i.e., there is only one inverse of a matrix
2. $(A^{-1})^{-1} = A$
3. $(kA)^{-1} = 1/kA^{-1}$
   Note: This is different from
   $(kA)^T = kA^T$
4. $(A^{-1})^T = (A^T)^{-1}$
5. $(A + B)^{-1} = A^{-1} + B^{-1}$
6. $(AB)^{-1} = B^{-1}A^{-1}$

Important things to note in Determinants

1. Determinant of Identity matrix = 1
   det (I) = 1

2. $|A^T| = |A|$

3. $|AB| = |A|\,|B|$

4. $|A^{-1}| = 1/|A|$

5. $|kA| = k^n |A|$ where n is order of matrix

6. Similarly,
   $|-A| = |-1 \times A|$
   $= (-1)^n \times |A|$

7. (adj A) A = A (adj) = |A|I

8. Deteminant of adj A
   $|\text{adj } A| = |A|^{n-1}$
   where n is the order of determinant