

Matrices and Determinants - Class XII

Past Year JEE Questions

Questions

Question: 01

Suppose A is any 3×3 nonsingular matrix and $(A - 3I)(A - 5I) = O$ where $I = I_3$ and $O = O_3$. If

$\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to :

- A. 8
- B. 7
- C. 13
- D. 12

Solutions

Solution: 01

Explanation

Given,

$$(A - 3I)(A - 5I) = O$$

$$\Rightarrow A^2 - 8A + 15I = O$$

Multiplying both sides by A^{-1} , we get,

$$A^{-1}A \cdot A - 8A^{-1}A + 15A^{-1}I = A^{-1}O$$

$$\Rightarrow A - 8I + 15A^{-1} = O$$

$$\Rightarrow A + 15A^{-1} = 8I$$

$$\Rightarrow \frac{A}{2} + \frac{15A^{-1}}{2} = 4I$$

Comparing with the equation $\alpha A + \beta A^{-1} = 4I$, we get

$$\alpha = \frac{1}{2} \text{ and } \beta = \frac{15}{2}$$

$$\therefore \alpha + \beta = \frac{1}{2} + \frac{15}{2} = \frac{16}{2} = 8$$