

## Matrices and Determinants - Class XII

### Past Year JEE Questions

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#### Questions

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##### Question: 01

If  $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ , then the matrix  $A^{-50}$  when  $\theta = \frac{\pi}{12}$  is equal to :

A.  $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

B.  $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$

C.  $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

D.  $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

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#### Solutions

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##### Solution: 01

##### Explanation

$$(A^{-50}) = (A^{-1})^{50}$$

We know,

$$A^{-1} = \frac{\text{adj}A}{|A|}$$

$$|A| = \cos^2\theta + \sin^2\theta = 1$$

$$\text{cofactor of } A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Adjoint of A = Transpose of cofactor matrix

$$\therefore \text{Adj } A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$\therefore (A^{-1})^2 = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

Similarly,

$$(A^{-1})^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$$

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$$(A^{-1})^{50} = \begin{bmatrix} \cos 50\theta & \sin 50\theta \\ -\sin 50\theta & \cos 50\theta \end{bmatrix}$$

when  $\theta = \frac{\pi}{12}$  then

$$A^{-50} = \begin{bmatrix} \cos \frac{25\pi}{6} & \sin \frac{25\pi}{6} \\ -\sin \frac{25\pi}{6} & \cos \frac{25\pi}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

Note:

$$\cos \frac{25\pi}{6} = \cos\left(4\pi + \frac{\pi}{6}\right) = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$