

Exemplar Problem

Matrix and Determinants

35. If x , y , z are all different from zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$, then value $x^{-1} + y^{-1} + z^{-1}$

is

A xyz

B $x^{-1} \cdot y^{-1} \cdot z^{-1}$

C $-x - y - z$

D -1

Ans: The correct answer is option *D*.

Here, we have $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$

Applying $C_1 \rightarrow C_1 - C_2$ and $C_2 \rightarrow C_2 - C_3$

$$\Rightarrow \begin{vmatrix} x & 0 & 1 \\ -y & y & 1 \\ 0 & -z & 1+z \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow x(y + yz + z) + yz = 0$$

$$\Rightarrow xy + xyz + xz + yz = 0$$

$$\Rightarrow xy + xz + yz = -xyz$$

$$\Rightarrow \frac{xy + xz + yz}{xyz} = -1$$

$$\Rightarrow \frac{1}{z} + \frac{1}{y} + \frac{1}{x} = -1$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = -1$$

$$\Rightarrow x^{-1} + y^{-1} + z^{-1} = -1$$

Hence, option *D* is the correct answer.