

## Exemplar Problem

### Matrix and Determinants

34. If  $A$  and  $B$  are invertible matrices, then which of the following is not correct?

$$A \operatorname{adj} A = |A| \cdot A^{-1}$$

$$B \det(A)^{-1} = [\det(A)]^{-1}$$

$$C (AB)^{-1} = B^{-1}A^{-1}$$

$$D (A + B)^{-1} = B^{-1} + A^{-1}$$

**Ans:** The correct answer is option  $D$ .

Since,  $A$  and  $B$  are invertible matrices. So, we can say that

$$(AB)^{-1} = B^{-1}A^{-1} \quad \text{it is correct}$$

$$\text{Also, } A^{-1} = \frac{1}{|A|}(\operatorname{adj} A)$$

$$\Rightarrow \operatorname{adj} A = |A| \cdot A^{-1} \quad \text{it is correct}$$

$$\text{And } AA^{-1} = I$$

$$\Rightarrow |AA^{-1}| = |I|$$

$$\Rightarrow |A| \cdot |A^{-1}| = 1$$

$$\Rightarrow |A^{-1}| = \frac{1}{|A|}$$

$$\Rightarrow \det(A)^{-1} = [\det(A)]^{-1} \quad \text{it is correct}$$

$$\text{Now, } (A + B)^{-1} = B^{-1} + A^{-1}$$

$$\text{L.H.S} = (A + B)^{-1}$$

$$= \frac{1}{|A + B|}(\operatorname{adj} A + \operatorname{adj} B)$$

$$\text{R.H.S} = B^{-1} + A^{-1}$$

$$= \frac{1}{|B|}(\operatorname{adj} B) + \frac{1}{|A|}(\operatorname{adj} A)$$

Thus, L.H.S  $\neq$  R.H.S *it is incorrect*

Hence, option  $D$  is the correct answer.