Example 16: The value of $\triangle = \begin{vmatrix} 1 & 1 & 1 \\ {}^{n}C_1 & {}^{n+2}C_1 & {}^{n+4}C_1 \\ {}^{n}C_2 & {}^{n+2}C_2 & {}^{n+4}C_2 \end{vmatrix}$ is 8.

Ans: Here, we have $\triangle = \begin{vmatrix} 1 & 1 & 1 \\ {}^{n}C_1 & {}^{n+2}C_1 & {}^{n+4}C_1 \\ {}^{n}C_2 & {}^{n+2}C_2 & {}^{n+4}C_2 \end{vmatrix}$

$$\Rightarrow \triangle = \begin{vmatrix} 1 & 1 & 1 \\ \frac{n!}{(n-1)!} & \frac{(n+2)!}{(n+1)!} & \frac{(n+4)!}{(n+3)!} \\ \frac{n!}{(n-2)!\,2!} & \frac{(n+2)!}{n!\,2!} & \frac{(n+4)!}{(n+2)!\,2!} \end{vmatrix}$$

$$\Rightarrow \triangle = \begin{vmatrix} 1 & 1 & 1 \\ \frac{n\ (n-1)!}{(n-1)!} & \frac{(n+2)(n+1)!}{(n+1)!} & \frac{(n+4)(n+3)!}{(n+3)!} \\ \frac{n(n-1)(n-2)!}{(n-2)!\,2!} & \frac{(n+2)(n+1)n!}{n!\,2!} & \frac{(n+4)(n+3)(n+2)!}{(n+2)!\,2!} \end{vmatrix}$$

$$\Rightarrow \triangle = \begin{vmatrix} 1 & 1 & 1 \\ n & (n+2) & (n+4) \\ \frac{n(n-1)}{2} & \frac{(n+2)(n+1)}{2} & \frac{(n+4)(n+3)}{2} \end{vmatrix}$$

Taking $\frac{1}{2}$ as common from R_3,

$$\Rightarrow \triangle = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ n & (n+2) & (n+4) \\ n(n-1) & (n+2)(n+1) & (n+4)(n+3) \end{vmatrix}$$

Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$, we get

$$\Rightarrow \triangle = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -4 & -2 & (n+4) \\ -8n-12 & -4n-10 & (n+4)(n+3) \end{vmatrix}$$

Now, expanding along R_1, we get

$$\Rightarrow \triangle = \frac{1}{2}\ [-4(-4n-10) + 2(-8n-12)]$$

$$\Rightarrow \triangle = \frac{1}{2}\ [16n + 40 - 16n - 24]$$

$$\Rightarrow \triangle = \frac{1}{2} \times 16$$

$\Rightarrow \triangle = 8$

Hence, the given statement is True.