

Exemplar Problem

Matrix and Determinants

Example 14: The value of the determinant

$$\Delta = \begin{vmatrix} \sin^2 23^\circ & \sin^2 67^\circ & \cos 180^\circ \\ -\sin^2 67^\circ & -\sin^2 23^\circ & \cos^2 180^\circ \\ \cos 180^\circ & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix} = \dots$$

Ans: Here, we have $\Delta = \begin{vmatrix} \sin^2 23^\circ & \sin^2 67^\circ & \cos 180^\circ \\ -\sin^2 67^\circ & -\sin^2 23^\circ & \cos^2 180^\circ \\ \cos 180^\circ & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}$

$$\Rightarrow \Delta = \begin{vmatrix} \sin^2 23^\circ & \sin^2 (90 - 23)^\circ & -1 \\ -\sin^2 67^\circ & -\sin^2 (90 - 67)^\circ & 1 \\ -1 & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} \sin^2 23^\circ & \cos^2 23^\circ & -1 \\ -\sin^2 67^\circ & -\cos^2 67^\circ & 1 \\ -1 & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \Delta = \begin{vmatrix} \sin^2 23^\circ + \cos^2 23^\circ - 1 & \cos^2 23^\circ & -1 \\ -(\sin^2 67^\circ + \cos^2 67^\circ) + 1 & -\cos^2 67^\circ & 1 \\ -1 + \sin^2 23^\circ + \sin^2 67^\circ & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 - 1 & \cos^2 23^\circ & -1 \\ -1 + 1 & -\cos^2 67^\circ & 1 \\ -1 + 1 & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & \cos^2 23^\circ & -1 \\ 0 & -\cos^2 67^\circ & 1 \\ 0 & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}$$

$\Rightarrow \Delta = 0$ All the elements of a column is 0

Hence, $\Delta = \begin{vmatrix} \sin^2 23^\circ & \sin^2 67^\circ & \cos 180^\circ \\ -\sin^2 67^\circ & -\sin^2 23^\circ & \cos^2 180^\circ \\ \cos 180^\circ & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix} = 0.$